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Clustering of Coupled Logistic Map with Bridge

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1. Introduction

In this study, we focus on "Bridge" in the network. The bridge is an edge which provide the only route between two nodes. Here, the edge is line connecting two nodes. The function of the bridge is called "Bridging function". In order to analyze complex phenomena of the bridge on the network, we use the coupled map. Generally, the coupled map is used as general models for complex dynamics, e.g. biological systems, economic activities and neural networks. Coupled oscillatory systems can also produce interesting phase patterns, e.g. "Clustering". In this study, we show synchronization phenomena of the network of coupled Logistic map when a coupling strength g and a non-liner parameter a are changed. Thereby, we observe influence of the bridge on the network. Moreover, we statistically investigate synchronized phenomena of the bridge.

2. Network model

We consider the network of coupled logistic map with the bridge. Network model including the bridge is shown in Fig. 1.

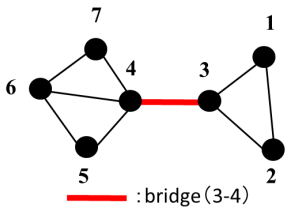


Figure 1: Network model including the bridge.

In Fig. 1, the bridge between 3 and 4 provides the only route which can flow information or influence from any contact of 3 to any contact of 4. The number of nodes and edges are set to 7 and 9. The coupling method of the logistic map is described as follows.

$$\begin{aligned} x_{i,n+1} &= (1-g)f(x_{i,n}) + \frac{g}{N_j}\sum_{j\in C_j} f(x_{j,n}), \\ i &= 1,2,3...,7, \end{aligned} \tag{1}$$

where x is the fast and slow dynamical variables, g is the coupling strength, C_j is set of nodes which are connected to node i, and N_j is the number of nodes which are connected to node i. We apply Logistic map to the function $f()$. Logistic map is described as follows.

$$x_{n+1} = ax_n(1-x_n), \tag{2}$$

where a is the non-liner parameter.

3. Simulation result

First, when a coupling strength g and a non-liner parameter a are changed, we show synchronization of the network of Logistic map which produce chaotic behavior. One of the simulation result is shown in Fig. 2. Here, two parameters are $a = 4.0$ and $g = 0.35$. In this simulation, iteration time is $n = 100000$. From Fig. 2, synchronization of bridge (3-4) is weak compared with other. Therefore, the bridge has a strong influence on network. Also, 1-2 and no-coupled 5-7 are full synchronization. Because these affected from same nodes.

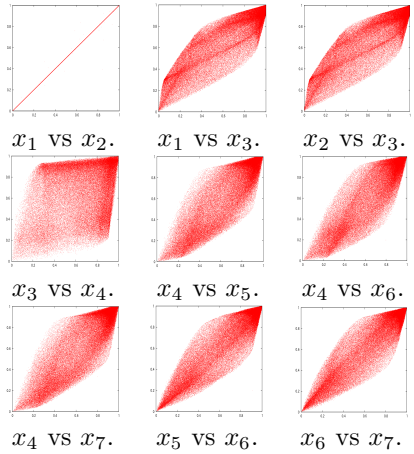
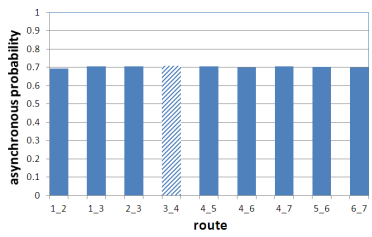
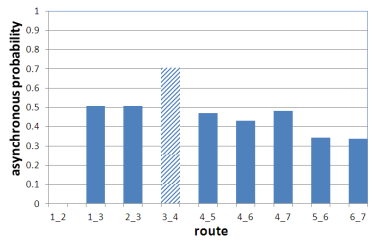


Figure 2: Simulation result. ($a = 4.0$, $g = 0.35$.)

Moreover, we statistically investigate synchronized phenomena of coupled Logistic map. Here, we define "Asynchronous" by $|x_i - x_j| > 0.1$. In this simulation, we define asynchronous probability from the number of "Corresponding" timing of bursting wave between two nodes for iteration time $n = 100000$. Asynchronous probability of simulation result is shown in Fig. 3. The parameters are same as Figs. 2 and 3 (b). When g is near 0, from Fig. 3(a), all maps are higher asynchronous probability. When g is increase, from Fig. 3(b), the bridge (3-4) has higher asynchronous probability than other route. Thereby, phase deviation occurs from the bridge statistically. Thus, we could confirm influence of the bridge on the network.



(a) $a = 4.0$, $g = 0.05$.



(b) $a = 4.0$, $g = 0.35$.

Figure 3: Asynchronous probability.

4. Conclusions

In this study, we have studied influence of the bridge in the network via synchronization phenomena of coupled Logistic map. From simulation results, in the network with a bridge, clustering of maps is occurred from a bridge. Moreover, we statistically confirmed that the bridge has a strong influence on the network.