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# Synchronization of Two Chaotic Circuits with LC Ladders Coupled by Resisters

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## 1. Introduction

In this study, we investigate two simple chaotic circuits with LC ladders coupled by resistors. By computer simulations, it is confirmed that the two chaotic circuits can be synchronized. Further, the existence of the solution on the in-phase synchronization can be explained theoretically.

## 2. Circuit Model

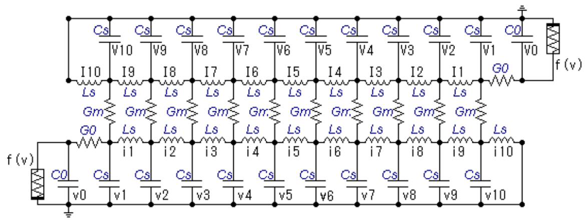


Figure 1: Circuit model ($G_m$: coupling resistor).

The chaotic circuit used in this study is a modified Chua's circuit [1]. We consider the case that the two modified Chua's circuits are placed in the opposite direction as shown in Fig. 1. We can derive the normalized state equations of this circuit as follows.

$$\begin{cases} \dot{x}_0 = -(x_0 - x_1) - g(x_0) \\ \dot{x}_k = \alpha\left\{\left(y_{(k-1)} - y_k\right) + \gamma_G\left(x_k - X_{(11-k)}\right)\right\} \\ \dot{y}_k = \beta\left(x_k - x_{(k+1)}\right) \\ \dot{X}_0 = -(X_0 - X_1) - g(X_0) \\ \dot{X}_k = \alpha\left\{\left(Y_{(k-1)} - Y_k\right) + \gamma_G\left(X_k - x_{(11-k)}\right)\right\} \\ \dot{Y}_k = \beta\left(X_k - X_{(k+1)}\right) \end{cases} \quad (1)$$

$$g(x) = bv - (a - b)\left\{|x - 1| - |x + 1|\right\}/2$$

where variables and parameters are defined by the following equations.

$$v_0 = Ex_0,\ V_0 = EX_0,\ v_k = Ex_k, V_k = EX_k,\ i_k = G_0Ey_k,\ I_k = G_0EY_k,$$
$$\alpha = C_0/C_S,\ \beta = C_0/(GL_S),\ \gamma_G = G_m/G_0,\ a = m_0/G_0,\ b = m_1/G_0,$$
$$t = G_0/C_0\tau,\ \text{“}\cdot\text{”} = d/d\tau.$$

## 3. Simulation results

We fix the parameters $\alpha = 0.5$, $\beta = 0.15$, $a = -1.25$, $b = -0.75$, and investigate the influence of the coupling parameters $\gamma_G$ on synchronization. In order to check the existence of synchronization states, we define the following dependent variables and substitute into Eq. (1).

$$\begin{cases} In-phase:\ \hat{x}_0 = x_0 - X_0,\ \hat{x}_k = x_k - X_k,\ \hat{y}_k = y_k - Y_k) \\ Anti-phase:\ \hat{x}_0 = x_0 + X_0,\ \hat{x}_k = x_k + X_k,\ \hat{y}_k = y_k + Y_k) \end{cases} \quad (2)$$

Next, the characteristic roots of the synchronization planes are investigated and these values are summarized in Fig. 2. Figure 3 shows some examples of computer simulation results. The left figures are the attractors and the right figures are the synchronization states between the two circuits.

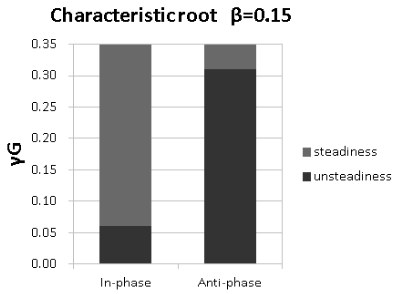


Figure 2: Characteristic roots.

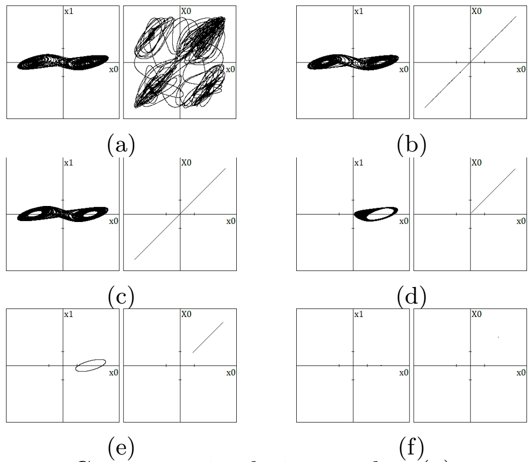


Figure 3: Computer simulation results. (a) $\gamma_G = 0.12$. (b) $\gamma_G = 0.13$. (c) $\gamma_G = 0.15$. (d) $\gamma_G = 0.20$. (e) $\gamma_G = 0.31$. (f) $\gamma_G = 0.32$.

For Figs. 3(b), (c), (d) and (e), the synchronization turned into a complete in-phase synchronization as indicated in Fig. 1. However, for Fig. 3(a), the in-phase synchronization plane does not seem to be stable on the contrary to the result in Fig. 1. For the $gamma_G$ value in Fig. 3(f), the attractor becomes point attractor, we could not confirm the stability of the synchronization planes. Additionally, as far as we carried out computer simulations, we could not confirm anti-phase synchronization of chaos although the existence of the stable region is confirmed theoretically.

## 4. Conclusions

In this study, we investigated the synchronization of two simple chaotic circuits with LC ladders coupled by resistors. By carrying out computer calculations, we observed chaos synchronizations.

**References**

[1] Yuki Nakaaji and Yoshifumi Nishio, “Synchronization of Chaotic Circuits with Transmission Line,” Proc. of NCSP'06, pp. 353-356, Mar. 2006.