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Synchronization Phenomena in van der Pol Oscillators with Difference Amplitudes

Vu Minh Thuan, Yoko Uwate and Yoshifumi Nishio
(Tokushima University)

1. Introduction

This paper gives an explanation of the synchronization phenomena in a circuit which contain three van der Pol Oscillators when one of them has different amplitude to the others.

2. Circuit Model

The circuit model is shown in Fig.1.

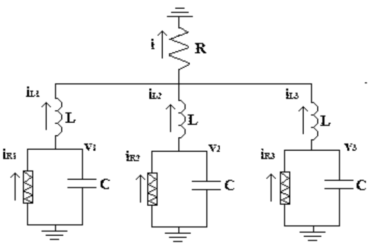

Figure 1: Circuit model.

The circuit equations are described as follows:

$$C\frac{dv_k}{dt} = -i_{Lk} - i_{Rk} \quad (1)$$

$$L\frac{di_{Lk}}{dt} = -v_k - R\sum_{j=1}^{3} i_{Lk} \quad (2)$$

where :

$$i_{Rk} = -g_1 v_k + g_3 {v^3}_k, \quad k = (1, 2) \quad (3)$$

$$i_{Rk} = \frac{1}{\beta}(-g_1 v_k + g_3 v_k^3), \quad (k = 3) \quad (4)$$

indicates v-i characteristics of nonlinear resistor, and β is the magnification of v_3's amplitude compared to v_1 and v_2's amplitude. We use Eq.(5)-(7) to change the variables as follows:

$$t = \sqrt{LC}\tau \quad (5)$$

$$v_k = \sqrt{\frac{g_1}{3g_3}}x_k, \; i_{Lk} = \sqrt{\frac{Cg_1}{3Lg_3}}y_k \quad (6)$$

$$\alpha = R\sqrt{\frac{C}{L}}, \quad \varepsilon = g_1\sqrt{\frac{L}{C}} \quad (7)$$

Equations (1),(2) is normalized as:

$$\frac{dx}{d\tau} = \varepsilon(x_k - \frac{1}{3}{x^3}_k)y_k, \quad (k = 1, 2) \quad (8)$$

$$\frac{dx}{d\tau} = \frac{\varepsilon}{\beta}(x_k - \frac{1}{3\beta^2}{x^3}_k)y_k, \quad (k = 3) \quad (9)$$

$$\frac{dx}{d\tau} = x_k - \alpha\sum_{j=1}^{3} y_j, \quad (k = 1, 2, 3) \quad (10)$$

In Eq.(7), α is the coupling factor and ε is the strength of nonlinearity.

3. Simulation Result

We used Runge-Kutta method to simulate Eqs.(8),(9), and obtained the results as follows:

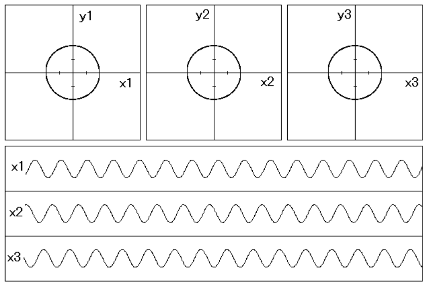

Figure 2: Simulation result when magnification β= 1.

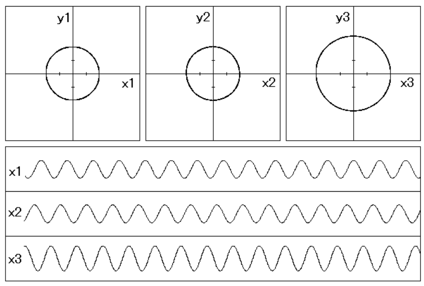

Figure 3: Simulation result when magnification β= 1.5.

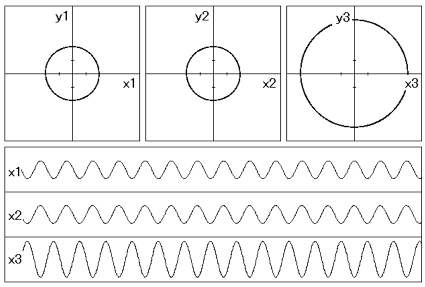

Figure 4: Simulation result when magnification β= 2.0.

Depend on what the magnification β is, we can observe the phase different between v_1, v_2, and v_3 shown in Figs. 2-4 above, in order to allow the current i is closed to zero. When the magnification $\beta = 1$, theoretically, we can see in Fig.2 that each phase is drifted 120 degree to the others, it seem to be happened in a three-phases generator, however when $\beta = 2$, phase of v_1 and v_2 is the same and reverse to phase of v_3.

4. Conclusions

This can be applied in some energy problem finding the lowest power consumption of a oscillators circuit.