Synchronization Phenomena in van der Pol Oscillators with Difference Amplitudes

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1. Introduction

This paper gives an explanation of the synchronization phenomena in a circuit which contain three van der Pol Oscillators when one of them has different amplitude to the others.

2. Circuit Model

The circuit model is shown in Fig.1.

![Circuit model](image)

Figure 1: Circuit model.

The circuit equations are described as follows:

\[
\begin{align*}
C_e \frac{dv_k}{dt} &= -i_{Lk} - i_{Rk} \quad (1) \\
L_e \frac{di_{Lk}}{dt} &= -v_k - R \sum_{j=1}^{3} i_{Lk} \quad (2)
\end{align*}
\]

where:

\[
\begin{align*}
i_{Rk} &= -g_1 v_k + g_3 v_3^k, \quad k = (1, 2) \quad (3) \\
i_{Rk} &= \frac{1}{\beta} (-g_1 v_k + g_3 v_3^k), \quad (k = 3) \quad (4)
\end{align*}
\]

indicates \(v-i\) characteristics of nonlinear resistor, and \(\beta\) is the magnification of \(v_3\)’s amplitude compared to \(v_1\) and \(v_2\)’s amplitude. We use Eq.(5)-(7) to change the variables as follows:

\[
\begin{align*}
t &= \sqrt{LC_0} \tau \quad (5) \\
v_k &= \sqrt{\frac{g_1}{3g_3}} x_k, \quad i_{Lk} = \sqrt{\frac{C_0 g_1}{3L_0 g_3}} y_k \quad (6) \\
\alpha &= R \sqrt{\frac{C}{L}}, \quad \varepsilon = g_1 \sqrt{\frac{L}{C}} \quad (7)
\end{align*}
\]

Equations (1),(2) is normalized as:

\[
\begin{align*}
\frac{dx}{dt} &= \varepsilon (x_k - \frac{1}{3} x^3_k) y_k, \quad (k = 1, 2) \quad (8) \\
\frac{dx}{dt} &= \frac{\varepsilon}{\beta} (x_k - \frac{1}{3} \beta^2 x^3_k) y_k, \quad (k = 3) \quad (9) \\
\frac{dx}{dt} &= x_k - \alpha \sum_{j=1}^{3} y_j, \quad (k = 1, 2, 3) \quad (10)
\end{align*}
\]

In Eq.(7), \(\alpha\) is the coupling factor and \(\varepsilon\) is the strength of nonlinearity.

3. Simulation Result

We used Runge-Kutta method to simulate Eqs.(8),(9), and obtained the results as follows:

![Simulation results](image)

Figure 2: Simulation result when magnification \(\beta= 1\).

![Simulation results](image)

Figure 3: Simulation result when magnification \(\beta= 1.5\).

![Simulation results](image)

Figure 4: Simulation result when magnification \(\beta= 2.0\).

Depend on what the magnification \(\beta\) is, we can observe the phase different between \(v_1\), \(v_2\), and \(v_3\) shown in Figs. 2-4 above, in order to allow the current \(i\) is closed to zero. When the magnification \(\beta = 1\), theoretically, we can see in Fig.2 that each phase is drifted 120 degree to the others, it seem to be happened in a three-phases generator, however when \(\beta = 2\), phase of \(v_1\) and \(v_2\) is the same and reverse to phase of \(v_3\).

4. Conclusions

This can be applied in some energy problem finding the lowest power consumption of a oscillators circuit.