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Synchronization of Two Rings with Different Sizes of Coupled Oscillators

Honami KUME Yoko UWATE Yoshifumi NISHIO
(Tokushima University)

1. Introduction

In this study, two rings of 5 and 7 van der Pol oscillators coupled by resistors are investigated. Both rings show 5-phase and 7-phase synchronizations when they are not coupled. Hence, the whole system becomes a frustrated network after coupling. Computer simulation results show some interesting synchronization phenomena.

2. Circuit Model

Figure 1 shows the circuit model. In the circuit, two rings of 5 and 7 van der Pol oscillators are coupled by resistors R'. Because the both rings have different sizes, 2 out of 7 oscillators in the lower ring have no connections with the upper ring. We assume that the characteristics

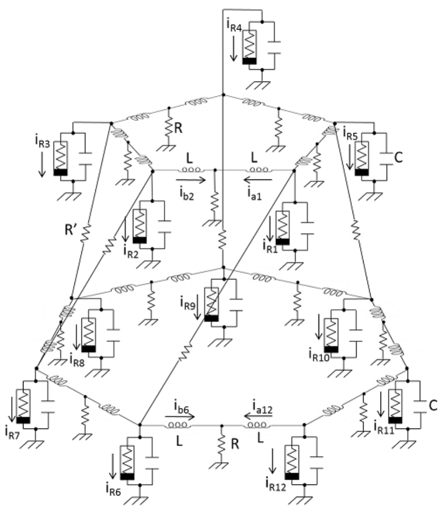

Figure 1: Circuit model.

of the nonlinear resistors are described by the following equation.

$$i_{Rk} = -g_1 v_k + g_3 v_k^3 \qquad (1)$$

The circuit equations are given as follows.

$$\frac{dx_k}{d\tau} = \begin{cases} \alpha(x_k - x_k^3) - (y_{ak} - y_{bk}) - \gamma(x_k - x_{(k+5)}) \\ \qquad : (k = 1 \sim 5) \\ \alpha(x_k - x_k^3) - (y_{ak} - y_{bk}) + \gamma(x_{(k-5)} - x_k) \\ \qquad : (k = 6 \sim 10) \\ \alpha(x_k - x_k^3) - (y_{ak} - y_{bk}) \\ \qquad : (k = 11, 12) \end{cases}$$
$$\frac{dy_{ak}}{d\tau} = x_k - \beta(y_{ak} + y_{b(k+1)})$$
$$\frac{dy_{bk}}{d\tau} = x_k - \beta(y_{a(k-1)} + y_{bk}) \qquad (2)$$

where

$$v_k = \sqrt{\frac{g_1}{g_3}} x_k, \quad i_{ak} = \sqrt{\frac{g_1}{g_3}} \sqrt{\frac{C}{L}} y_{ak}, \quad i_{bk} = \sqrt{\frac{g_1}{g_3}} \sqrt{\frac{C}{L}} y_{bk},$$

$$t = \sqrt{LC}\tau, \quad \alpha = g_1 \sqrt{\frac{L}{C}}, \quad \beta = R\sqrt{\frac{C}{L}}, \quad \gamma = \frac{1}{R'}\sqrt{\frac{L}{C}}. \qquad (3)$$

3. Simulation Results

Figures 2 and 3 show typical examples of computer simulation results for the cases of weak and strong couplings between the rings, respectively. For the case of the weak coupling, as shown in Fig. 2, 5-phase synchronization and 7-phase synchronization are observed from the upper and the lower rings, respectively. However, for the strong coupling, as shown in Fig. 3, the different synchronization states of the two rings affect each other and hence the amplitudes of some of the oscillators become smaller and the phase differences of the rings are not complete 5-phase or 7-phase anymore.

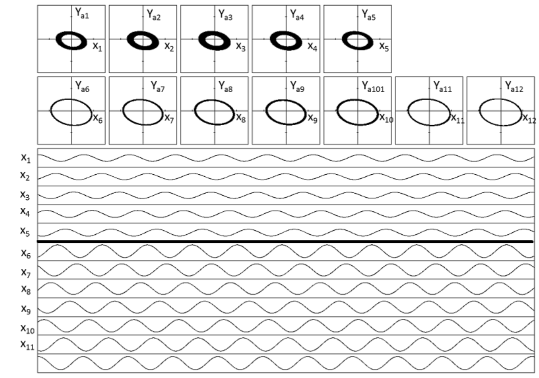

Figure 2: Attractors and time wave forms for weak coupling. α=0.1, β=1.1, and γ=0.01.

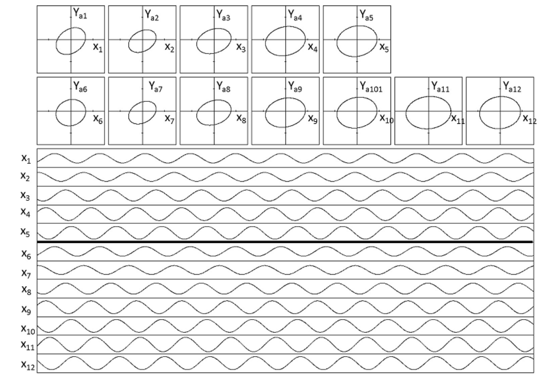

Figure 3: Attractors and time wave forms for strong coupling. α=0.1, β=1.1, and γ=10.

4. Conclusions

In this study, we have investigated synchronization phenomena observed from two rings with different sizes of coupled van der Pol oscillators. We observed attractors and time wave forms by computer simulations. We could confirm frustrations between the rings.

Theoretical analysis of the synchronizations is our future research.