Investigation of Clustering and Local Bridges in a Complex Network by Coupled Rulkov Maps

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***Abstract*—It is important to understand dynamics of complex networks. We focus on network with "local bridge" which is one of the complex networks. In this study, we investigate influence of the local bridge for network using synchronization phenomena of coupled Rulkov maps. From simulation results, when the maps switch from full synchronization to clustering, phase deviation occurs from the local bridges.**

I. Introduction

We focus on social network which is one of complex network. Understanding dynamics of the social network is important because "social network analysis" has been used not only for analyzing modern society but also for physics, biology and information science by many researchers [1]-[4]. The social network is structure which shows social relation, e.g. aerial line, infection with a virus, World Wide Web and so on.

In this study, we focus on "local bridge" which is found by M. Granovetter [5]. We need to explain keyword "bridge" before explaining the local bridge. The bridge is an edge which provides the only route between two nodes. Here, edge is line connecting two nodes. Example of the bridge is shown in Fig. 1.

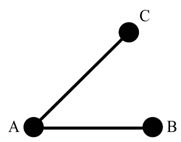


Fig. 1. Example of the bridge.

Here, the bridge between A and B provides the only route which can flow information or influence from any contact of A to any contact of B. The function of the bridge is called "bridging function". The bridge happens only rarely in large network actually. However, in the case of large scale network, the bridging function may be provided locally. Example of the local bridge is shown in Fig. 2.

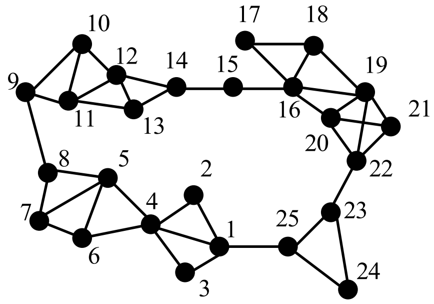


Fig. 2. Example of the local bridge.

In Fig. 2, the shortest route from 1 to 25 is 1-25. One of second shortest route from 1 to 25 is 1-4-5-8-9-11-13-14-15-16-20-22-23-25. The 1-25 route is not strictly bridge because other routes construct. However, the 1-25 route is predominantly shorter than other routes. In this case, the bridging function is provided locally by the 1-25 route and this bridge is called the local bridge. Similarly, it is found that 8-9, 14-15, 15-16 and 22-23 route are local bridge. In the social network, the local bridge affects the entire network in terms of propagation of information. Because the local bridge creates more and shorter routes.

In order to analyze complex phenomena of the social network with local bridge, we use coupled maps [7]. Generally, the coupled maps is used as general models for the complex dynamics, e.g. biological systems, economic activities and neural network. Coupled oscillatory systems can also produce interesting phase patterns, "clustering" and complex phase patterns. Moreover, a discrete map for spiking and spiking-bursting neural behavior was proposed by Rulkov [6]. Rulkov maps can be useful for understanding the dynamical mechanism of oscillators in "the large scale network". In this study, we use Rulkov maps applying for the coupled maps. Because the social network has features of cluster and the large scale network, and we consider that the propagation of information on the social network is realized by the Rulkov maps applying for the coupled maps.

In this study, we show synchronization of the social network of Rulkov maps when a coupling strength g and a control parameter α are changed. Thereby, we observe the influence of the local bridge on the social network.

II. Social Network of Coupled Rulkov Maps

We consider the case of the social network with local bridge. In this study, this social network is shown in Fig. 2. The equation of the Rulkov maps is described as follows.

$$\begin{aligned} x_{i,n+1} &= f(x_{i,n}, x_{i,n-1}, y_{i,n}) + \frac{g}{N}\sum_{j\in C_j}(x_{j,n} - x_{i,n}), \\ y_{i,n+1} &= y_{i,n} - \mu(x_{i,n}+1) + \mu\sigma_i + \mu\frac{g}{N}\sum_{j\in C_j}(x_{j,n} - x_{i,n}), \\ i &= 1, ..., N, \end{aligned} \quad (1)$$

where x and y are the fast and slow dynamical variables, g is the coupling strength, N is the number of nodes in the social network, $\mu = 0.001$. C_j is set of nodes which are connected to node i. The function $f()$ has the following form:

$$f(x_n, y_n) = \begin{cases} \alpha/(1-x_n) + y_n, & x_n \leq 0 \\ \alpha + y_n, & 0 < x_n < \alpha + y_n \ and \ x_{n-1} \leq 0 \\ -1, & x_n \geq \alpha + y_n \ or \ x_{n-1} > 0, \end{cases} \quad (2)$$

Here, we determine α_0 which is standard value. The control parameter α of each cell is $\alpha_0 + \Delta\alpha$. $\Delta\alpha$ is set for randomly distributed in the interval $[0 : \Delta\alpha]$.

A. Synchronization Phenomena of Social Network

First, when the coupling strength g and the control parameter α are changed, we show synchronization phenomena of the social network of Rulkov maps which produce spiking behavior. These behaviors are shown in Fig. 3.

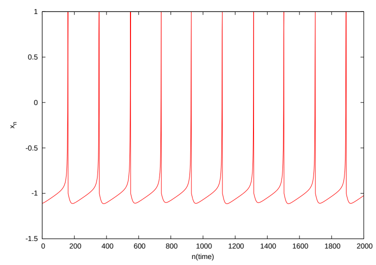
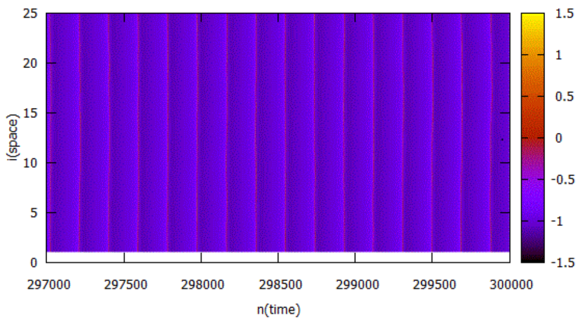
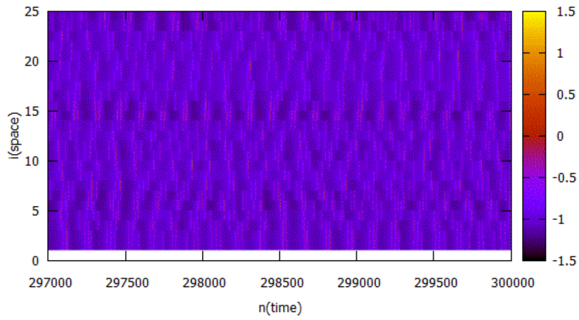


Fig. 3. Typical wave forms of Rulkov maps of Spiking behavior.

In this simulation, we define two synchronization phenomena for spiking behavior. First, "full synchronization" is that all maps are synchronized at the in-phase. Second, "clustering" is that maps is synchronized at the in-phase locally. The simulation results are shown in Fig. 4. The horizontal axis is iteration time n. The vertical axis is space i. The color of Fig. 4 is the dynamical variable x. The simulation results are shown in Fig. 4. Here, we determine that the standard value α_0 is 4.0 and σ is 0.01. From Fig. 4 (a), the maps show full synchronization at steady state if the coupling strength g is high. On the other hand, from Fig. 4 (b), the maps show



(a) $g = 0.03$, $\Delta\alpha = 0$.



(b)$g = 0.001$, $\Delta\alpha = 0.5$.

Fig. 4. Space-time diagram for spiking behavior. (a) Full synchronization. (b) Asynchronization.

asynchronization at steady state if the coupling strength g is low and the control parameter α vary with each node.

However, from Fig. 5 (a), the maps switch from full synchronization and clustering at steady state. When the maps switch from full synchronization to clustering, phase deviation occurs from the local bridge 22-23. From Fig. 5 (b), the maps show full synchronization and clustering even if the parameters are changed. Moreover, the maps show more cluster than Fig. 5 (a). Phase deviation occurs from the local bridges even if the maps show many clusters. Thereby, the local bridge has a strong influence on network.

III. Conclusion

In this study, we have studied influence of the local bridge in the social network via synchronization phenomena of Rulkov maps applying for the coupled maps. From simulation results, when the maps switch from full synchronization to clustering, phase deviation occurs from the local bridges.

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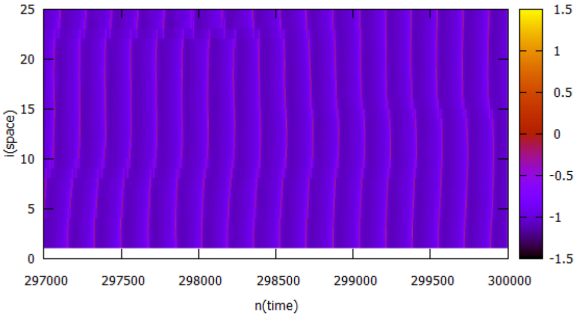
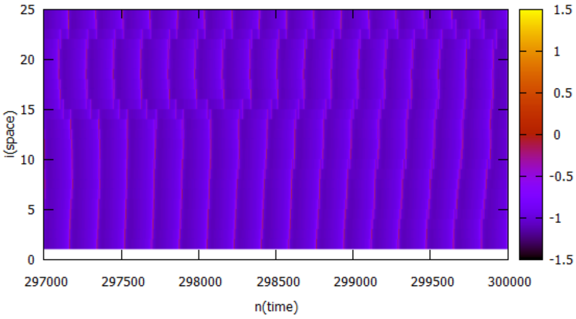


(a) $g = 0.0252$, $\Delta\alpha = 0.06$.



(b) $g = 0.009$, $\Delta\alpha = 0.03$.

Fig. 5. Space-time diagram by changing parameters for spiking behavior.

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