Synchronization of Oscillatory Ring Topology Coupled by Two Types of Resistors

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***Abstract*— In this study, we investigate synchronization phenomena observed in coupled oscillatory networks by using two types of resistors. Negative and normal resistors are used as coupling resistors in our circuit model. By giving particular parameters, we can observe the oscillation death and amplification of the amplitude. Finally we extend coupled oscillators to polygonal oscillatory networks.**

I. Introduction

There are a lot of synchronization phenomena in this world. This is one of the nonlinear phenomena that we can often observe by natural beings which does collective actions. For example, firefly luminescence, cry of birds and frogs, applause of many people and so on. Synchronization phenomena have a feature that the set of small power can produce very big power by synchronizing at a time. Therefore studies of synchronization phenomena have been widely reported not only engineering but also the physical and biological fields [1]-[6]. Investigation of coupled oscillators attracts attention from many researchers because coupled oscillatory network produce interesting phase synchronization such as the phase propagation wave, clustering and complex patterns.

In our investigations, we use several van der Pol oscillators (see Fig. 1(a)). Van der Pol oscillators have been coupled in various form and investigated about their synchronization phenomena [7], [8]. We have observed three-phase synchronization in the circuit model which are shown in Fig. 1(b) since the discrepancy occurs in synchronization phenomena between adjacent oscillators [9]. This three-phase synchronization is always observed stability.

On the other hand, we can observe generically in-phase synchronization in two coupled oscillators for Fig. 2(a) and anti-phase synchronization in the case of Fig. 2(b). Namely what kind of synchronization phenomena will be observed in coupled oscillators which are included these two types of synchronization states? First, we consider coupled oscillators including two types of coupling resistor which is shown in Fig. 3. Seeing partly, anti-phase synchronization is observed between first and second oscillators and in-phase synchronizations are observed between first and third or second and third oscillators. However because frustration occurs in whole system, we guess unexpected synchronization phenomena are found out.

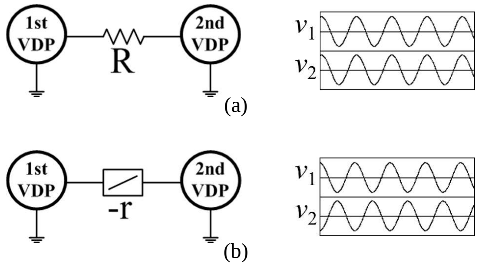

Fig. 2. Synchronization phenomena between first and second oscillators. (a) In-phase synchronization. (b) Anti-phase synchronization.

II. Synchronization Phenomena

In this study, we investigate synchronization phenomena of coupled oscillatory networks using different types of resistors

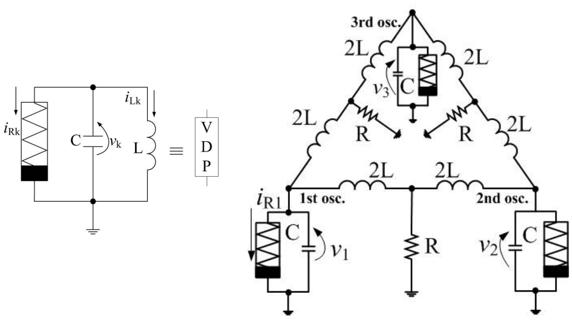

Fig. 1. (a) Van der Pol oscillator. (b) Three coupled oscillators.

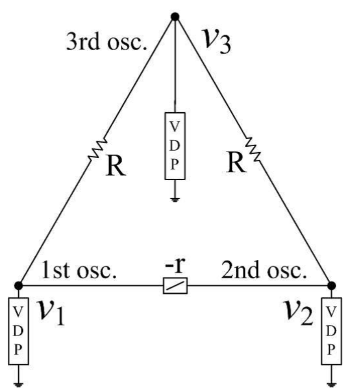

Fig. 3. Conceptual circuit model for three coupled oscillators.

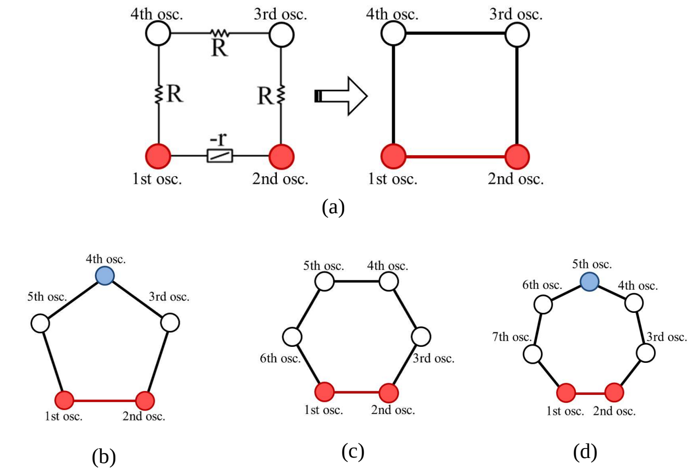

Fig. 4. Several types of coupled polygonal oscillatory networks. (a) tetragon network. (b) pentagon network. (c) hexagon network. (d) heptagon network.

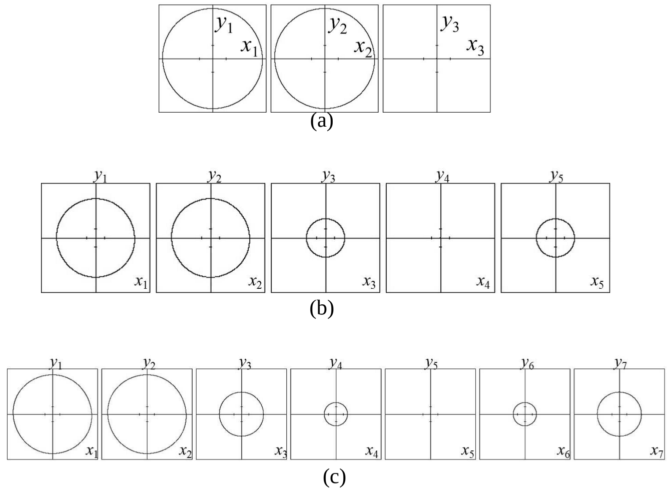

Fig. 5. Simulation results for polygonal oscillatory networks. (a) triangle network for $\gamma_1 = \gamma_2 = 0.050$. (b) pentagon network for $\gamma_1 = \gamma_2 = 0.050$. (c) heptagon network for $\gamma_1 = \gamma_2 = 0.075$.

partially. We focus on the amplitude of each oscillator and phase differences between adjacent oscillators by changing coupling strength. Also, we increase number of oscillators in the ring topology and we consider differences of synchronization phenomena by changing number of oscillators.

III. Synchronization Phenomena

In this section, we show the simulation result for several types of oscillatory networks as shown Fig. 4. We calculate circuit equations using the fourth-order Runge-Kutta method with the step size $h = 0.001$. We fix the parameter $\varepsilon = 0.020$ in these simulations. First, we explain the synchronization phenomena in the case of triangle networks for coupling strength $\gamma_1 = \gamma_2 = 0.050$. We show the simulation result of each oscillator in Fig. 5(a). Obvious difference of the amplitude of each oscillator appears in this parameters. Namely, oscillation death is observed in third oscillator. Phase differences between first and second oscillators shows anti-phase synchronization, because third oscillator stops oscillating in order to match consistency.

In the case of polygonal oscillatory networks, different synchronization phenomena for odd and even number are produced even if same parameter are given. For example, giving same values for coupling strength, odd polygonal oscillatory networks cause oscillation death. The oscillator which oscillation stops is the farthest oscillator from first and second oscillators. We show the farthest oscillator as blue oscillator in Fig. 4(b) and (d). Also, getting away first and second oscillators, oscillators attenuate. Figure 5(b) and (c) show simulation results for odd polygonal networks and we set coupling strength $\gamma_1 = \gamma_2 = 0.050$ for pentagon network and $\gamma_1 = \gamma_2 = 0.075$ for heptagon network. In the case of heptagon network, fifth oscillator attenuates but does not occur the oscillation death for $\gamma_1 = \gamma_2 = 0.050$.

On the other hand, even polygonal oscillator networks can not produce oscillation death but we can observe small and large of amplitudes. Regardless of the number of oscillators, synchronization state between first and second oscillators is observed anti-phase synchronization and case of between other oscillators are observed in-phase synchronization except the farthest oscillator.

IV. Conclusion

In this study, we have been able to observe several patterns of synchronization phenomena. For three coupled oscillators, we have observe the oscillation death in third oscillator. In addition, we have investigated differences of synchronization phenomena in the ring topology by increasing number of oscillators. We could confirm that odd polygonal networks produced oscillation death but even one do not produce oscillation death.

In our future works, we will derive each amplitude for polygonal networks by using averaging method.

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