1-30

Synchronization for Social Network with Local Bridge using Coupled Rulkov Maps

Tomoya Shima†, Yoko Uwate†, Thomas Ott‡ and Yoshifumi Nishio†
(†Tokushima University), (‡Zurich University of Applied Sciences)

1. Introduction

In this study, we focus on "local bridge" in social network. Example of network including the local bridge is shown in Fig. 1.

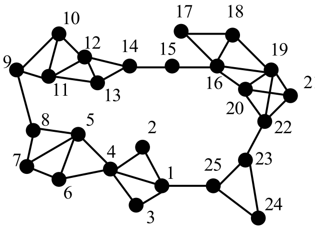


Figure 1: Example of network including the local bridge.

In Fig. 1, the shortest route from 1 to 25 is 1-25. One of second shortest route from 1 to 25 is 1-4-5-8-9-11-13-14-15-16-20-22-23-25. The 1-25 route is predominantly shorter than other routes. In this case, the 1-25 route is called the local bridge. In the social network, the local bridge affects the entire network in terms of propagation of information. In order to analyze complex phenomena, we use coupled maps. Generally, the coupled maps is used as general models for the complex dynamics, e.g. biological systems. Coupled oscillatory systems can also produce interesting phase patterns, "clustering". Moreover, a discrete map for spiking-bursting neural behavior was proposed by Rulkov [1]. Rulkov maps can be useful for understanding the dynamical mechanism of oscillators in "large scale network". In this study, we use Rulkov maps applying for the coupled maps because the social network has features of clustering and large scale network.

2. Social network of Rulkov maps

We consider the social network (see. Fig. 1) of coupled maps. The number of coupled map is set to 25. We describe equations based on a chain of coupled maps.

$$x_{i,n+1} = f(x_{i,n}, x_{i,n-1}, y_{i,n}) + \frac{g}{N}\sum_{j\in C_j}(x_{j,n} - x_{i,n}),$$

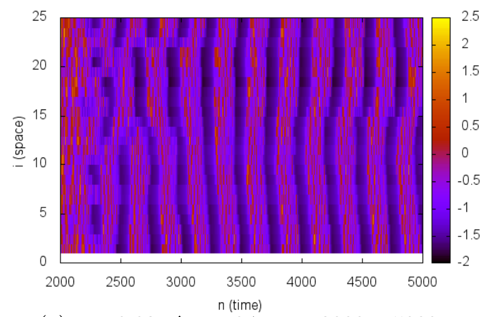
$$y_{i,n+1} = y_{i,n} - \mu(x_{i,n}+1) + \mu\sigma_i + \mu\frac{g}{N}\sum_{j\in C_j}(x_{j,n} - x_{i,n}),$$

$$i = 1, ..., N, \qquad (1)$$

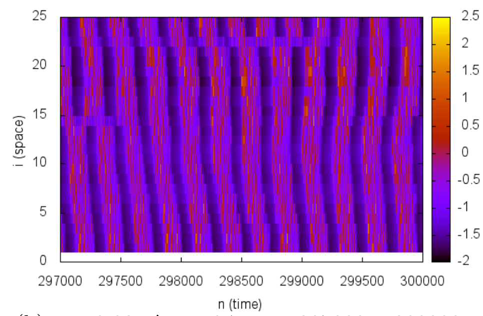
where x and y are the fast and slow dynamical variables, g is the coupling strength, N is the number of cells in the social network, $\mu = 0.001$, $\sigma = 0.15$, C_j is set of nodes which are connected to node i. The function $f()$ has the following form:

$$f(x_n, y_n) = \begin{cases} \alpha/(1-x_n) + y_n, x_n \le 0 \\ \alpha + y_n, 0 < x_n < \alpha + y_n \; and \\ \qquad x_{n-1} \le 0 \\ -1, x_n \ge \alpha + y_n \; or \; x_{n-1} > 0. \end{cases} \qquad (2)$$

Next, we show synchronization of the social network of Rulkov maps which produce spiking-bursting behavior. In this study, we define two maps as synchronization if timing of bursting wave between two maps is right. Here, "full synchronization" is that all maps are synchronized at the in-phase, and "clustering" is that maps is synchronized at the in-phase locally. The simulation results are shown in Fig. 2. The horizontal axis is iteration time n and the vertical axis is space i. The complex propagation wave can be observed when g is 0.03 and $\Delta\alpha_{max}$ is 0.7 (see. Fig. 2). From Fig. 2 (a), we assume that the maps tend to converge full synchronization by repeating update. However, in the case of Fig. 2 (b), the maps switch between full synchronization and clustering. Also, when the maps switch from full synchronization to clustering, phase deviation occurs from the local bridges. In the case of Figs. 2 (a) and (b), the phase deviation occurs from local bridge 14-15 and 22-23. From these result, in the social network with local bridge, the social network of Rulkov maps does not converges full synchronization. Because the maps switch between full synchronization and clustering by the local bridge.



(a) $g = 0.03$, $\Delta\alpha = 0.7$, $n = 2000 - 5000$.



(b) $g = 0.03$, $\Delta\alpha = 0.7$, $n = 297000 - 300000$.

Figure 2: Propagation wave for the social network.

3. Conclusions

In this study, we have studied influence of the local bridge in the social network via synchronization phenomena of Rulkov maps applying for the coupled maps. From simulation results, in the social network with local bridge, the maps switch between synchronization and clustering by the local bridge.

References

[1] S. Mark, "The Strength of Weak Ties," *Amerian Journal of Sociology*, vol 78, pp 1360-1380, 1973.