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Two Types of Waves in a Ladder of Simultaneous Oscillators Coupled by Inductors

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1. Introduction

In the natural fields, various synchronization phenomena exist. For example, firefly luminescence, swing of pendulums, cardiac heartbeat, and so on, are well known as synchronization phenomena. In this study, we investigate a ladder of simultaneous oscillators with two LC resonators coupled by inductors. By computer simulation, we observe two types of waves.

2. Circuit Model

The circuit model is shown in Fig. 1. In the circuit, twenty simultaneous oscillators with two LC resonators are coupled by inductors L_C and each simultaneous oscillator consists of a nonlinear negative resistor, whose $v-i$ characteristics are described by a fifth-power polynomial function as

$$i_R(v) = g_1 v - g_3 v^3 + g_5 v^5 \qquad (g_1,\ g_3,\ g_5 > 0), \qquad (1)$$

and two resonators with different natural angular frequencies ($\sqrt{L_1C_1}$ and $\sqrt{L_2C_2}$). The equations governing the coupled oscillators are described by the following differential equations including twenty nonlinear functions i_{Rj} ($j = 1, 2, ..., 20$).

$$\begin{cases} \dfrac{dx_{j1}}{d\tau} = -y_{j1} - f(x_{j1}+x_{j2}) - y_{Cj} + y_{C,j-1} \\ \dfrac{dx_{j2}}{d\tau} = \alpha_C\{-y_{j2} - f(x_{j1}+x_{j2}) - y_{Cj} + y_{C,j-1}\} \\ \dfrac{dy_{j1}}{d\tau} = x_{j1} \\ \dfrac{dy_{j2}}{d\tau} = \alpha_L x_{j2} \qquad (j = 1, 2, ..., 20), \end{cases} \qquad (2)$$

and y_{C0} and $y_{C,20}$ are zero; and the nonlinear function $f(\cdot)$ which corresponds to the $v-i$ characteristics of the nonlinear resistors is given as

$$f(x) = \varepsilon\left(x - \frac{\beta}{3}x^3 + \frac{1}{5}x^5\right) \ . \qquad (3)$$

3. Simulation Results

In this article, we show only several computer simulated results obtained by giving different initial conditions for the fixed parameters as $\alpha_C = 0.51$, $\alpha_L = 1.0$, $\gamma = 0.01$, $\varepsilon = 0.98$ and $\beta = 2.73$. As a result, we can discover two types of waves. First wave is a propagation change of phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase. An example of this type of wave is shown in Fig. 2(a). This figure shows time waves of the sum of the voltages of two horizontally adjacent resonators. We call this wave as "phase inversion waves". In this figure, black area of the diagram shows in-phase synchronization states, while white area shows anti-phase synchronization states or oscillation death. Second wave is a propagation change of oscillation states from oscillation death to oscillation or from oscillation to oscillation death. We call this wave as "oscillatory inversion wave". An example of this type of wave is shown in Fig. 2(b). This figure shows time waves of the voltages of each resonators. Hence, white area simply shows the oscillation death. In this figure, we can confirm that the upper resonators oscillate only when the lower resonators stop to oscillate, and vice verse.

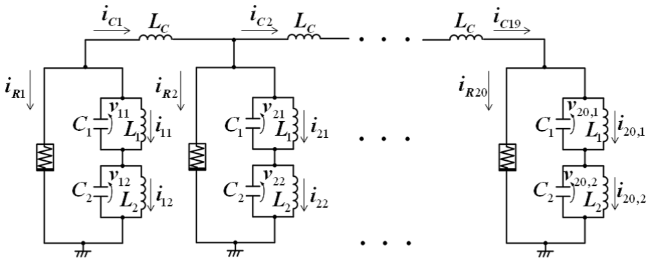

Figure 1: Twenty simultaneous oscillators with two resonators coupled by inductors.

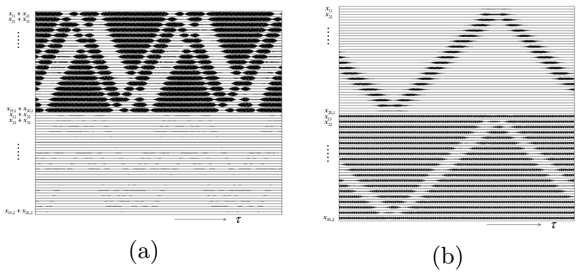

Figure 2: Time waves of voltages. (a) Upper: $x_{j1,1}+x_{j+1,1}$ and Lower: $x_{j2}+x_{j+1,2}$. (b) Upper: x_{j1} and Lower: x_{j2}.

4. Conclusions

In this study, we have investigated the circuit model that twenty simultaneous oscillators with two resonators are coupled by inductors L_C. We could observe two types of waves. First wave is a propagation change of phase states between two adjacent oscillators from in-phase to anti-phase or from anti-phase to in-phase. Second wave is a propagation change of oscillation states from oscillation death to oscillation or from oscillation to oscillation death.