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# Switching of Phase Differences in Coupled Chaotic Circuits with Regular Tetrahedron Form

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## 1. Introduction

Coupled chaotic systems are suitable models to express many kinds of high-dimensional nonlinear phenomena. In [1], coupled chaotic circuits have produced anti-phase synchronization or chaotic synchronization. In this study, we investigate synchronization phenomena in coupled chaotic circuits with regular tetrahedron form.

## 2. Circuit Model

We show the circuit diagram of coupled chaotic circuits in Fig. 1. Each chaotic circuit is composed of two inductors, a capacitor, a negative resistor and diodes. We couple chaotic circuits via $L_1$ and ground by coupling resistor $R$ in this study. In the computer simulations, we assume that the $v_d - i_k$ characteristics of the nonlinear resistor are given by diodes.

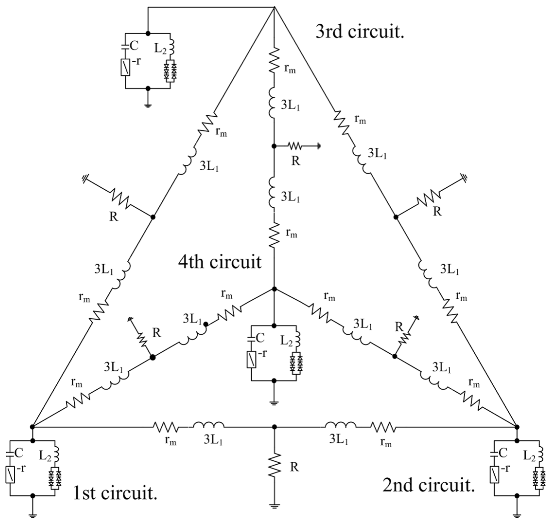


Figure 1: Circuit model in tetrahedron form.

The normalized circuit equations are expressed as:

$$\begin{cases} \dfrac{dx_{ak}}{d\tau} = \dfrac{1}{3}\Big\{(x_{ak}+x_{bk}+x_{ck}+y_k) - z_k - \gamma(x_{ak}+x_n)\Big\} \\ \dfrac{dx_{bk}}{d\tau} = \dfrac{1}{3}\Big\{(x_{ak}+x_{bk}+x_{ck}+y_k) - z_k - \gamma(x_{bk}+x_n)\Big\} \\ \dfrac{dx_{ck}}{d\tau} = \dfrac{1}{3}\Big\{(x_{ak}+x_{bk}+x_{ck}+y_k) - z_k - \gamma(x_{ck}+x_n)\Big\} \\ \dfrac{dy_k}{d\tau} = \alpha\beta(x_{ak}+x_{bk}+x_{ck}+y_k) - z_k - f(y_k) \\ \dfrac{dz_k}{d\tau} = x_{ak}+x_{bk}+x_{ck}+y_k. \end{cases} \tag{1}$$

where

$$I_k = a\sqrt{\frac{C}{L_1}}x_k\ ,\ i_{ak} = a\sqrt{\frac{C}{L_1}}y_{ak}\ ,\ i_{bk} = a\sqrt{\frac{C}{L_1}}y_{bk},$$

$$i_{ck} = a\sqrt{\frac{C}{L_1}}y_{ck}\ ,\ v_k = az_k\ ,\ t = \sqrt{L_1C}\tau\ ,$$

$$\alpha = \frac{L_1}{L_2},\ \beta = r\sqrt{\frac{C}{L_1}},\ \gamma = R\sqrt{\frac{C}{L_1}},\ a = \sqrt[8]{r_d\sqrt{\frac{C}{L_1}}}$$

(k= 1, 2, 3, 4),

and

$$f(y_k) = \sqrt[9]{y_k}. \tag{2}$$

$\beta$ is bifurcation parameter, $\gamma$ is coupling strength and $y_n$ denotes the current of adjacent oscillator.

## 3. Synchronization Phenomena

We calculate Eq. (1) using the fourth-order Runge-Kutta method with the step size $h = 0.001$. In this simulation, we fix the parameter $\alpha = 20.0$ and change the bifurcation parameter or the coupling strength. As a result, we can find out very interesting phase synchronizations which could not been observed in four coupled van der Pol oscillators. For example, we can observe the four-phase synchronization or the phase shift which occurs alternately anti-phase synchronization and asynchronous.

We show one example of the synchronization phenomena in Fig. 2. We set parameters as $\beta = 0.330$ and $\gamma = 0.280$. This figure shows that synchronization states between the first and the other circuits change with time. The red point shows the synchronization phenomena between the first and the second circuits. Namely, this means that phase differences are switching in a particular aria by effects of adjacent chaotic circuits.

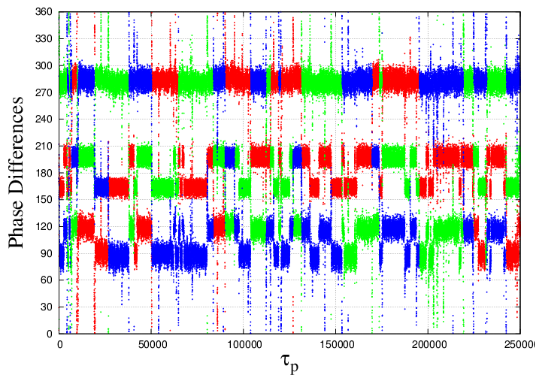


Figure 2: Phase shifts for $\beta = 0.330$ and $\gamma = 0.280$ (red is phase difference between 1st-2nd circuit, green is between 1st-3rd, blue is between 1st-4th).

## 4. Conclusions

This article presents synchronization phenomena of coupled chaotic circuits in regular tetrahedral form. In this circuit model, we have been able to observe several patterns of synchronization phenomena.

## References

[1] Y. Nishio, K. Suzuki, S. Mori and A. Ushida, "Synchronization in Mutually Coupled Chaotic Circuits," *Proc. of ECCTD'93*, vol. 1, pp. 637-642, Aug. 1993.