Assessment of Stability for Oscillatory Circuit by Using Spice

Hiroshige Kataoka, Yoko Uwate, Yoshihiro Yamagami and Yoshifumi Nishio
Department of Electrical and Electronic Engineering
Tokushima University
Tokushima, Japan
Email: {hiroshige, uwate, yamagami, nishio} @ee.tokushima-u.ac.jp

***Abstract*— SPICE is a very convenient tool for circuit simulation and is used by many researchers. Nowadays, various SPICE-oriented algorithm are proposed. By using these methods, we can extend a function of SPICE and can analyze various circuit.**

In this study, we propose a SPICE-oriented algorithm for assessment of stability for oscillatory circuit. We combine the harmonic balance method, Newton homotopy method and Floquet theory. We find out a oscillatory parameter by using harmonic balance method and Newton homotopy method, and we assess the stability by applying our method. As an example, we assess the stability of the periodic solutions for Cauer oscillator. The result shows our propose method gives the correct results.

I. Introduction

SPICE is used for various analysis of the electrical circuit. For example, AC analysis, DC analysis, sensitivity analysis, transient analysis etc. In addition, we can easily perform a SPICE simulation. We only have to make a netlist or schematic by computer, without writing a complex program. From this reason, many people use SPICE for circuit simulation. On the other hand, many researchers have proposed SPICE simulation method, which is the introduction of analytical dynamics. We propose a SPICE-oriented algorithm by applying the analytical dynamics and combine to the conventional method.

For designing oscillatory circuit, it is important to the assessment of the stability. We have proposed a SPICE-oriented algorithm to the assessment of the stability for periodic solutions which is based on the Floquet theory [1]. In the conventional method, we assessed the stability of resonant circuit. In this study, we apply the our method to the oscillatory circuit by combining the Newton homotopy method [2]-[4].

The article is organized as follows. Section II-*A* shows how to use the sine-cosine circuits [5], which is based on the HB (harmonic balance) [6]-[8] method. We use the sine-cosine circuit to obtain the value of the voltages which are required in order to solve variational circuits. Section II-*B* shows the Newton homotopy method. This method is realized by using solution-curve tracing circuit(STC) [9]. Section II-*C* shows the Floquet theory [8][10]. Section III shows an illustrative example and how to solve the variational circuits by using SPICE. Section IV shows the results and confirms the effectiveness of the proposed method. Section V concludes this article.

II. SPICE-Oriented analysis of oscillator

A. Sine-Cosine Circuit

The sine-cosine circuit has been introduced in order to solve the determining equations of the harmonic balance method by using SPICE. In this section, we explain how we can derive the sine-cosine circuit for simple passive element cases.

First, we set a voltage and a current with Fourier series;

$$\begin{cases} v = V_0 + \sum_{k=1}^{n}(V_{s_k}\sin k\omega t + V_{c_k}\cos k\omega t) \\ i = I_0 + \sum_{k=1}^{n}(I_{s_k}\sin k\omega t + I_{c_k}\cos k\omega t) \end{cases} \quad (1)$$

A current through a capacitor is given by

$$i = C\frac{dv}{dt}. \quad (2)$$

From Eqs. (1) and (2), we can express the current as

$$i = \sum_{k=1}^{n}(-k\omega C V_{c_k}\sin k\omega t + k\omega C V_{s_k}\cos k\omega t). \quad (3)$$

From Eq. (3), we can express the relation between the coefficients of sine and cosine components as follows;

$$\begin{cases} I_{s_k} = -k\omega C V_{c_k} \\ I_{c_k} = k\omega C V_{s_k} \end{cases} \quad (4)$$

In the case of an inductor, we can express the voltage across an inductor as

$$v = \sum_{k=1}^{n}(-k\omega L I_{c_k}\sin k\omega t + k\omega L I_{s_k}\cos k\omega t), \quad (5)$$

where the current through an inductor is given by

$$v = L\frac{di}{dt}. \quad (6)$$

Equations for coefficient of $\sin k\omega t$ and $\cos k\omega t$ are given by

$$\begin{cases} V_{s_k} = -k\omega L I_{c_k} \\ V_{c_k} = k\omega L I_{s_k} \end{cases}. \quad (7)$$

If we make the circuit model satisfying this method, a capacitor is replaced by coupled voltage-controlled current sources and an inductor is replaced by coupled current-controlled voltage sources in the sine-cosine circuit.

B. Newton Homotopy Method

Newton homotopy method is one of method for finding multiple dc solutions. The circuit model of Newton homotopy method is shown in Fig. 1. We assume equations as follows;

$$\begin{cases} g_0(V_0, V_1, V_2, \ldots, V_M) = 0 \\ g_1(V_0, V_1, V_2, \ldots, V_M) = 0 \\ g_2(V_0, V_1, V_2, \ldots, V_M) = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ g_{M-1}(V_0, V_1, V_2, \ldots, V_M) = 0 \\ g_M(V_0, V_1, V_2, \ldots, V_M) = 0 \end{cases} . \tag{8}$$

These determining equations are described by a set of algebraic equations, which consists of M-equations and same number of variables. However, it is not easy to solve the equations, because they may have the multiple solutions.

Applying the Newton homotopy method to solve Eq. (8), we obtain the following relation;

$$\boldsymbol{G}(\boldsymbol{V}, \rho) = \boldsymbol{g}(\boldsymbol{V}) - (1-\rho)\boldsymbol{g}(\boldsymbol{V_{(0)}}) = \boldsymbol{0}. \tag{9}$$

where the initial state is set by a point $(\boldsymbol{V_{(0)}}, \rho = 0)$ and gets the solutions satisfying $g(v) = 0$ at $\rho = 1$ on the path. ρ shows solutions curves called homotopy paths, and find the multiple solutions lying on the paths. A solution curve is traced by ark-length method as follows;

$$\begin{cases} \boldsymbol{G}(\boldsymbol{V}, \rho) = \boldsymbol{0} \\ \sum^{M} \left(\frac{dV_i}{ds}\right)^2 + \left(\frac{d\rho}{ds}\right)^2 = 1 \\ i = 1 \\ i \neq 2 \end{cases} . \tag{10}$$

Equation (10) is realized by using ABMs. Figure 2 shows the circuit diagram of solution-curve tracing circuit (STC).

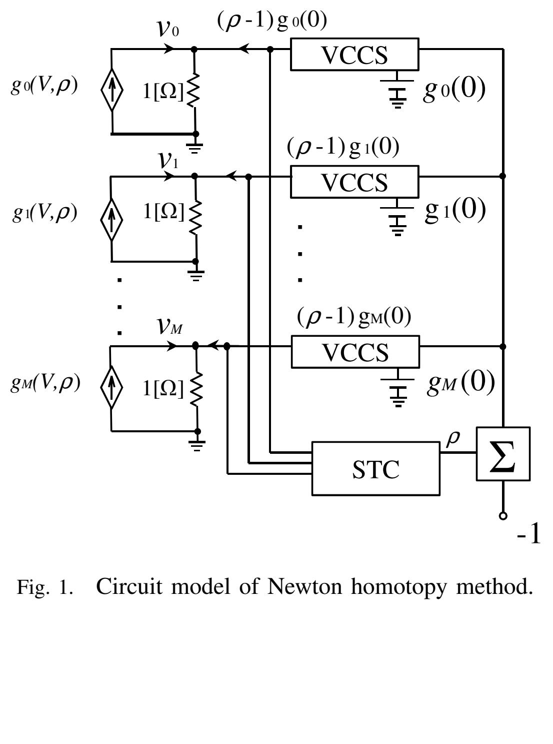

Fig. 1. Circuit model of Newton homotopy method.

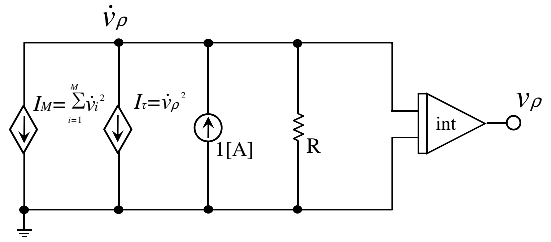

Fig. 2. Solution-curve tracing circuit (STC).

C. Stability of Periodic Solutions

We suppose that there is a circuit equation as

$$f(\dot{x},\ x,\ y,\ \omega t) = 0, \tag{11}$$

and make the variational equation for the regular period solution of $\hat{x}$. First, we assume the small change quantity as $(\Delta x, \Delta y)$ as

$$\begin{cases} x = \hat{x} + \Delta x \\ y = \hat{y} + \Delta y \end{cases} , \tag{12}$$

and substitute Eq. (12) to Eq. (11). We obtain the equation as

$$f(\dot{\hat{x}},\ \hat{x},\ \hat{y},\ \omega t) + \left[\frac{\partial f}{\partial \dot{x}} \frac{\partial f}{\partial x} \frac{\partial f}{\partial y}\right] |_{x=\hat{x}, y=\hat{y}} \begin{bmatrix} \dot{\Delta x} \\ \Delta x \\ \Delta y \end{bmatrix} = 0. \tag{13}$$

In Eq. (13), the first term is regular period solution and second term is variational equation. We change the second term as

$$\dot{\Delta x} = A(t)\Delta x. \tag{14}$$

In Eq. (14), $A(t)$ is the periodic function. We apply the Floquet theory for this periodic function. We write the Jacobian matrix of the periodic solution as $\Phi(t)$. From this, the solution after one period from initial value of $\Delta x(0)$ is given as follows;

$$\Delta x(T) = \Phi(T)\Delta x(0). \tag{15}$$

Hence, when the eigenvalues $(\lambda_1, \lambda_2, \ldots, \lambda_n)$ of $\Phi(T)$ satisfy $|\lambda_k| < 1$ $(k = 1, 2, \ldots, n)$, the regular periodic solution $\hat{x}$ is stable.

In this study, we derive the variational circuit which corresponds to the variational equation and perform the transient analysis of Spice just for one period in order to obtain the components of $\Phi(T)$. We should repeat the transient analysis by giving different initial conditions to obtain all the components numerically. However, the number of the repeat is at most the same as the number of the state variables of the circuit. Further, it should be mentioned that we do not have to change the structure of the variational circuit even when the voltages of the regular periodic solution are changed.

III. Illustrative example

As an illustrative example, we assess the stability with the circuit in Fig. 3. The circuit parameters in Fig. 3 are set as $\alpha = \beta = 1$, $R_1 = R_2 = R_3 = 0.1$, $L_1 = 0.4167$, $L_2 = 0.2618$, $L_3 = 1.058$, $C_1 = 0.1$, $C_2 = 0.3429$ and $C_3 = 0.439$.

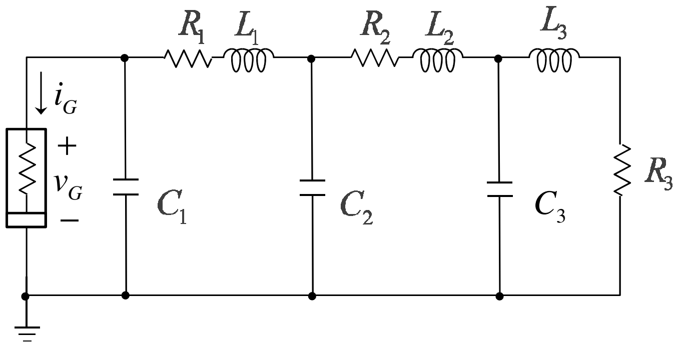

Fig. 3. Cauer oscillator.

The circuit equation can be written as

$$\begin{cases} C_1\dfrac{dv_1}{dt} + i_1 = \alpha v_1 - \beta {v_1}^3 \\ i_1 = i_2 + C_2\dfrac{dv_2}{dt} \\ i_2 = i_3 + C_3\dfrac{dv_3}{dt} \end{cases} . \tag{16}$$

If we write the variables as periodic solutions with small variations;

$$\begin{cases} i_k = i_{k0} + \Delta i_k \\ v_k = v_{k0} + \Delta v_k \end{cases} \tag{17}$$

We obtain the following variational equations as

$$\begin{cases} C_1\dfrac{dv_{10}}{dt} + \Delta i_1 = \alpha \Delta v_1 - 3\beta {v_{10}}^2 \Delta v_1 \\ \Delta i_1 = \Delta i_2 + C_2\dfrac{\Delta dv_2}{dt} \\ \Delta i_2 = \Delta i_3 + C_3\dfrac{\Delta dv_3}{dt} \end{cases} , \tag{18}$$

where we neglect higher-order small terms. From these equations, we can make the variational circuit of Fig. 3 as shown in Fig. 4.

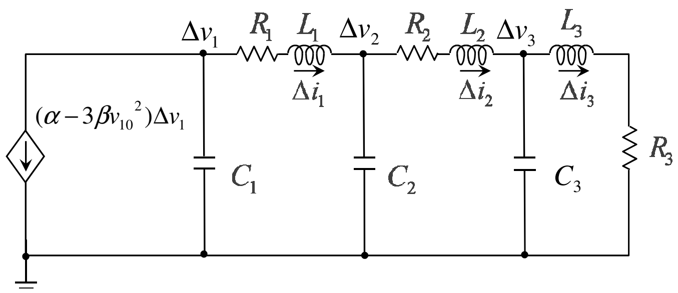

Fig. 4. Variational circuit of Fig. 3.

In Fig. 4, v_{10} is a steady periodic solution and are calculated as

$$v_{10} = V_c \cos\omega t + V_s \sin\omega t, \tag{19}$$

where V_c and V_s are given by the sine-cosine circuit obtained from the circuit in Fig. 3.

IV. Simulation Results

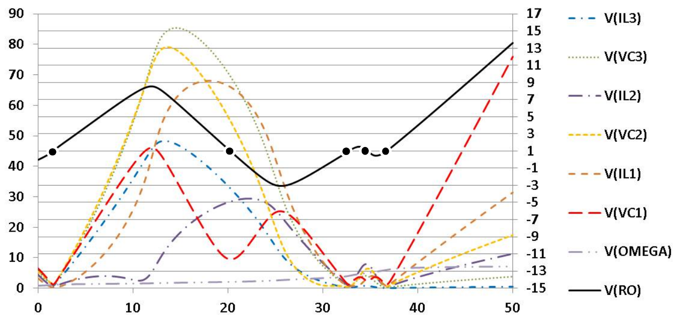

Fig. 5. Result of transient analysis in Fig.4.

Figure 5 is the time-response obtained by solving the circuit in Fig. 3 with HB method combining with Newton homotopy method. We found 5 equilibrium points, where ρ (=V(RO)) satisfying $\rho = 1$. We show the solutions in Table I.

TABLE I
Eigenvalues obtained by conventional method

time [sec]	1.5728	20.26	32.72	34.723	36.694
V(Omega)	0.993	2.003	4.002	4.999	5.9797
V(VC1)	1.211	9.446	1.216	2.346	1.1918
V(IL1)	0.1563	65.945	0.499	3.339	0.716
V(VC2)	1.1520	54.84	0.416	6.410	0.598
V(IL2)	0.520	28.398	1.03	7.853	0.513
V(VC3)	1.009	69.43	0.677	3.9160	0.208
V(IL3)	0.956	32.727	0.160	0.740	0.033

We give the obtained value to the circuit of Fig. 4 as initial condition. The state of initial condition are given as follows;

$$(\Delta v_{10}, \Delta i_{10}, \Delta v_{20}, \Delta i_{20}, \Delta v_{30}, \Delta i_{30}) \tag{20}$$

$$= \begin{cases} (\mathrm{V(VC1)}, 0, 0, 0, 0, 0) \\ (0, \mathrm{V(IL1)}, 0, 0, 0, 0) \\ (0, 0, \mathrm{V(VC2)}, 0, 0, 0) \\ (0, 0, 0, \mathrm{V(IL2)}, 0, 0) \\ (0, 0, 0, 0, \mathrm{V(VC3)}, 0) \\ (0, 0, 0, 0, 0, \mathrm{V(IL3)}) \end{cases} . \tag{21}$$

We perform the transient analysis with the circuit in Fig. 4 just for one period and obtain the components of $\Phi(T)$ as the values of the state variables of the variational circuit. We show $\Phi(T)$ for the 5 cases in Table II. Table III shows eigenvalues of $\Phi(T)$ for 5 cases, which calculated by MATLAB. We can see that the point $\omega = 0.993$, $\omega = 4.0022$ and $\omega = 5.9797$ are stable, because all of eigenvalues satisfy $|\lambda| < 1$. However, for the other two points, $\omega = 2.0028$ and $\omega = 4.999$, the solutions are unstable, because some eigenvalues do not satisfy $|\lambda| < 1$. These results agree with the results obtained in [4]. Namely, we can say that our algorithm gives the correct results.

TABLE II
THE VALUES AFTER ONE PERIOD $\Phi(T)$

(a) ω=0.993

Δv_1	Δi_1	Δv_2	Δi_2	Δv_3	Δi_3
1.74m	-9.31m	838.9μ	4.08μ	1.67m	-1.01m
-5.06m	27.07m	-1.07m	172.8μ	-5.09m	2.79m
2.76m	-6.56m	-181.00m	-38.07m	33.00m	18.88m
1.06μ	368.49μ	-13.33m	90.25m	4.13m	-2.55m
6.16m	-34.57m	37.01m	13.00m	-122.78m	6.98m
-8.51m	43.34m	48.33m	-19.05m	15.57m	8.71m

(b) ω=2.0028

Δv_1	Δi_1	Δv_2	Δi_2	Δv_3	Δi_3
8.97μ	-2.41m	-33.81μ	-3.16m	229.98μ	573.26μ
-107.69m	28.93	368.68m	37.94	-2.66	-6.89
-1.04m	269.11m	10.02	1.12	-26.39	1.07
-38.26m	10.28	488.71m	-8.97	573.54m	1.85
11.41m	-3.03	-42.77	2.41	-15.55	-464.38m
32.32m	-8.68	1.93	8.60	-542.72m	18.68

(c) ω=4.0022

Δv_1	Δi_1	Δv_2	Δi_2	Δv_3	Δi_3
-2.11m	11.89m	-13.55m	-3.05m	14.41m	-16.17m
20.57m	-115.25m	120.45m	44.94m	-140.34m	150.30m
-16.04m	82.74m	74.55m	-247.64m	103.94m	-24.97m
-7.00m	58.45m	-468.93m	107.90m	203.78m	119.56m
35.59m	-200.25m	216.63m	224.77m	272.48m	-95.58m
-22.70m	122.12m	-29.50m	74.93m	-54.25m	19.11m

(d) ω=4.999

Δv_1	Δi_1	Δv_2	Δi_2	Δv_3	Δi_3
-133.99μ	2.42m	-6.39m	12.28m	-5.72m	-11.89m
14.93m	-269.94m	705.24m	-1.32	579.10m	1.28
-61.98m	1.11	-2.17	282.53m	103.94m	-24.97m
111.37m	-1.95	248.94m	5.25	143.10m	800.73m
-43.34m	720.03m	2.63	116.79m	59.38m	-803.17m
-41.11m	722.05m	-349.93m	304.82m	-365.69m	190.94m

(e) ω=5.9797

Δv_1	Δi_1	Δv_2	Δi_2	Δv_3	Δi_3
-279.42μ	1.40m	-369.76μ	12.82m	-24.20m	-15.52m
3.53m	-17.92m	12.36m	-192.80m	300.38m	209.46m
-642.02μ	8.29m	-127.32m	356.41m	130.64m	-149.45m
14.70m	-87.00m	233.17m	186.32m	-115.71m	61.94m
-18.74m	92.17m	58.22m	-78.73m	21.73m	-49.11m
-4.58m	24.44m	-25.35m	16.01m	-18.70m	12.03m

V. CONCLUSION

We proposed the SPICE-oriented algorithm to assess the stability of periodic solutions for oscillator. We obtained periodic solutions by using harmonic balance method and Newton homotopy method. We assessed the stability based on the Floquet theory. In detail, we analyzed the Cauer oscillator for 5 different frequencies which gives both stable and unstable solutions. Our results agree well with the previously obtained results. For the next step of our research, we need to work on assessment of stability of oscillator having complex characteristics.

REFERENCES

[1] H. Kataoka, Y. Yamagami and Y. Nishio, “SPICE-Oriented Algorithm for Assessment of Stability for Periodic Solutions,” Proc. of NOLTA’10, pp.374-377, 2010.

[2] Y. Inoue, “DC Analysis of Nonlinear Circuits Using Solution-Tracing Circuits,” Trans. IEICE, vol.J74-A, pp.1647-1655, 1991.

TABLE III
EIGENVALUES OF $\Phi(T)$.

ω	0.993	2.003	4.002	4.999	5.980
λ_1	-0.202	-38.947	-0.411	5.670	0.402
λ_2	-0.112	0	0.467	-4.154	-0.359
λ_3	0.093	-18.424	0.365	-1.717	0.181
λ_4	0.034	18.120	-0.186	0	-0.004
λ_5	0	39.230	0	2.889	0.050
λ_6	0.101	33.133	0.122	0.885	-0.196

[3] A. Ushida, Y. Yamagami, I. Kinouchi, Y. Nishio and Y. Inoue, “An Efficient Algorithm for Finding Multiple DC Solutions Based on the SPICE-Oriented Newton Homotopy Method,” IEEE Trans. Computer-Aided Design of Integrated Circuits Syst., vol.21, no.3, pp.337-348, Mar. 2002.

[4] Y. Yamagami, Y. Nishio and A. Ushida, “Analysis of Reactance oscillators Having Multi-Mode Oscillations,” IEICE Trans. Fundamentals., vol.E89-A, no.3 Mar. 2006.

[5] J. Kawata, Y. Taniguchi, M. Oda, Y. Yamagami, Y. Nishio and A. Ushida, “SPICE-Oriented Frequency-Domain Analysis of Nonlinear Electronic Circuits,” IEICE Trans. Fundamentals, vol.E90-A, no.2, Feb. 2007.

[6] P. Wambacq and W. Sansen, *Distortion Analysis of Analog Integrated Circuits*, Kluwer Academic Publishers, 1979.

[7] D.O. Pederson and K. Mayaram, *Analog Integrated Circuits for Communication*, Kluwer Academic Publishers, 1991.

[8] A.H. Nayfeh and D.T. Mook, *Nonlinear Oscillations*, Wiley-Inter Science, 1998.

[9] E. Ikeno and A. Ushida, “The Arc-Length Method for the Computation of Characteristic Curves,” IEEE Trans. Circuits Syst., vol.23, pp.181-183, 1976.

[10] F.L. Traversa, F. Bonani and S.D. Guerrieri, “A Frequency-Domain Approach to the Analysis of Stability and Bifurcations in Nonlinear Systems Described by Differential-Algebraic Equations,” Int. J. Circuit Theory and Appl., vol.36, pp.421-439, 2008.