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Double-Mode Simultaneous Oscillation in Coupled Hard Oscillators

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***Abstract*—In this study, synchronization phenomenon observed from two inductively coupled simultaneous oscillators can exhibit both in-phase and anti-phase synchronizations. We can confirm the phenomenon that double-mode and simultaneous oscillation is generated at the same time.**

I. Introduction

In 1954, Schaffner reported that an oscillator with two degrees of freedom could oscillate simultaneously at two different frequencies when the nonlinear characteristics are described by a fifth-power polynomial function [1]. Kuramitsu investigated the simultaneous oscillations for three or more degrees case theoretically and confirmed the generation of triple mode oscillation by circuit experiments [2]. Simultaneous oscillations are definitely one of the most common nonlinear phenomena observed in various higher-dimensional systems in the natural fields. However, after their pioneering works, as far as the authors know, there have not been many researches clarifying the basic mechanism of the simultaneous oscillations except [3][4].

In our past study, we have reported synchronization phenomena observed from two resistively coupled simultaneous oscillators [5] and two inductively coupled simultaneous oscillators [6]. In this study, we investigate two inductively coupled simultaneous oscillators with more degrees of freedom. We confirm the generation of various interesting modes of oscillations from the circuits.

II. Circuit model

The circuit model is shown in Fig. 1. In the circuit, two simultaneous oscillators with three LC resonators are coupled by an inductor L_C and each simultaneous oscillator consists of a nonlinear negative resistor, whose $v-i$ characteristics are described by a fifth-power polynomial function as

$$i_R(v) = g_1 v - g_3 v^3 + g_5 v^5 \qquad (g_1, g_3, g_5 > 0) \tag{1}$$

and three resonators with different natural frequencies. The equations governing the coupled oscillators are described by the following 12th-order differential equations including two nonlinear functions i_{R1} and i_{R2}.

$$\begin{cases} C_1\dfrac{dv_{11}}{dt} = -i_{11} - i_{R1} - i_C, & L_1\dfrac{di_{11}}{dt} = v_{11} \\ C_2\dfrac{dv_{12}}{dt} = -i_{12} - i_{R1} - i_C, & L_2\dfrac{di_{12}}{dt} = v_{12} \\ C_3\dfrac{dv_{13}}{dt} = -i_{13} - i_{R1} - i_C, & L_3\dfrac{di_{13}}{dt} = v_{13} \\ C_1\dfrac{dv_{21}}{dt} = -i_{21} - i_{R2} + i_C, & L_1\dfrac{di_{21}}{dt} = v_{21} \\ C_2\dfrac{dv_{22}}{dt} = -i_{22} - i_{R2} + i_C, & L_2\dfrac{di_{22}}{dt} = v_{22} \\ C_3\dfrac{dv_{22}}{dt} = -i_{23} - i_{R2} + i_C, & L_3\dfrac{di_{23}}{dt} = v_{23} \end{cases} \tag{2}$$

where i_C is the current through the coupling inductor and is given as

$$i_C = \frac{L_1(i_{11} - i_{21}) + L_2(i_{12} - i_{22}) + L_3(i_{13} - i_{23})}{L_C} \tag{3}$$

The currents through the nonlinear resistors i_{R1} and i_{R2} are given as

$$\begin{cases} i_{R1} = i_R(v_{11} + v_{12} + v_{13}) \\ i_{R2} = i_R(v_{21} + v_{22} + v_{23}) \end{cases} \tag{4}$$

By using the following variables and parameters,

$$\begin{gathered} v_{mn} = \sqrt[4]{\frac{g_1}{5g_5}}x_{mn}, \quad i_{mn} = \sqrt[4]{\frac{g_1}{5g_5}}\sqrt{\frac{C_1}{L_1}}y_{mn}, \\ \alpha_{C1} = \frac{C_1}{C_2}, \quad \alpha_{L1} = \frac{L_1}{L_2}, \quad \gamma = \frac{L_1}{L_C}, \\ \alpha_{C2} = \frac{C_1}{C_3}, \quad \alpha_{L2} = \frac{L_1}{L_3}, \\ \varepsilon = g_a\sqrt{\frac{L_1}{C_1}}, \quad \beta = \frac{3g_3}{g_1}\sqrt{\frac{g_1}{5g_5}}, \quad t = \sqrt{L_1C_1}\tau, \end{gathered} \tag{5}$$

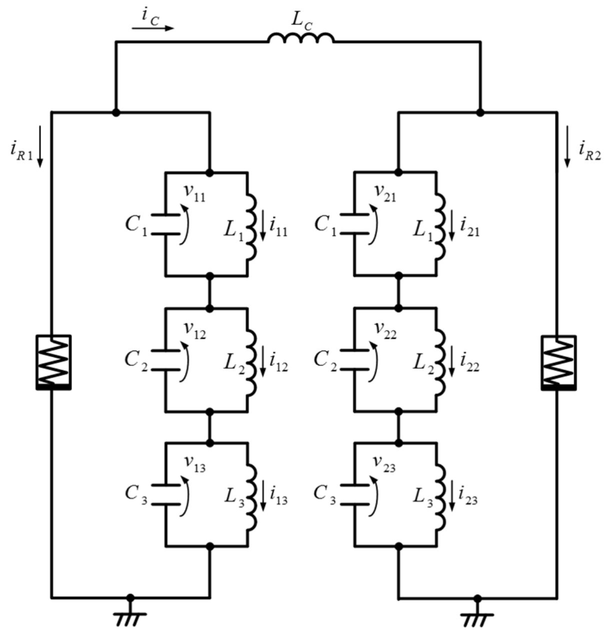

Fig. 1. Circuit diagram.

the normalized circuit equations are given as follows.

$$\begin{cases} \dfrac{dx_{11}}{d\tau} = -y_{11} - f(x_{11} + x_{12} + x_{13}) - y_C \\ \dfrac{dy_{11}}{d\tau} = x_{11} \\ \dfrac{dx_{12}}{d\tau} = \alpha_{C1}\{-y_{12} - f(x_{11} + x_{12} + x_{13}) - y_C\} \\ \dfrac{dy_{12}}{d\tau} = \alpha_{L1} x_{12} \\ \dfrac{dx_{13}}{d\tau} = \alpha_{C2}\{-y_{13} - f(x_{11} + x_{12} + x_{13}) - y_C\} \\ \dfrac{dy_{13}}{d\tau} = \alpha_{L2} x_{13} \\ \dfrac{dx_{21}}{d\tau} = -y_{21} - f(x_{21} + x_{22} + x_{23}) - y_C \\ \dfrac{dy_{21}}{d\tau} = x_{21} \\ \dfrac{dx_{22}}{d\tau} = \alpha_{C1}\{-y_{22} - f(x_{21} + x_{22} + x_{23}) - y_C\} \\ \dfrac{dy_{22}}{d\tau} = \alpha_{L1} x_{22} \\ \dfrac{dx_{23}}{d\tau} = \alpha_{C2}\{-y_{23} - f(x_{21} + x_{22} + x_{23}) + y_C\} \\ \dfrac{dy_{23}}{d\tau} = \alpha_{L2} x_{23} \end{cases} \tag{6}$$

where y_C corresponds to i_C and is given as

$$y_C = \gamma\left(y_{11} - y_{21} + \frac{y_{12} - y_{22}}{\alpha_{L1}} + \frac{y_{13} - y_{23}}{\alpha_{L2}}\right) \tag{7}$$

and the nonlinear function $f(\cdot)$ which corresponds to the $v-i$ characteristics of the nonlinear resistors is given as

$$f(x) = \varepsilon\left(x - \frac{\beta}{3}x^3 + \frac{1}{5}x^5\right) \tag{8}$$

III. Synchronization Phenomena

We investigate the phenomenon by changing the parameters.

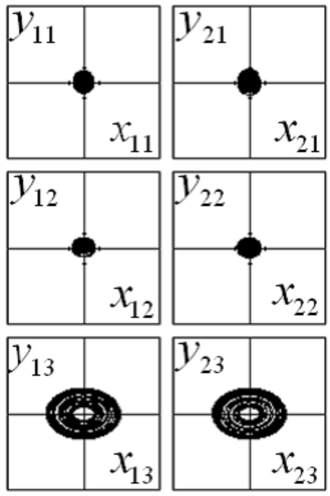

Fig. 2. Lissajous figures of six resonators. Horizontal: x_{mn}. Vertical: y_{mn}. $\alpha_{C1} = 0.5$, $\alpha_{L1} = 0.3$, $\alpha_{C2} = 0.4$, $\alpha_{L2} = 0.2$, $\gamma = 0.01$, $\varepsilon = 0.055$ and $\beta = 3.14$.

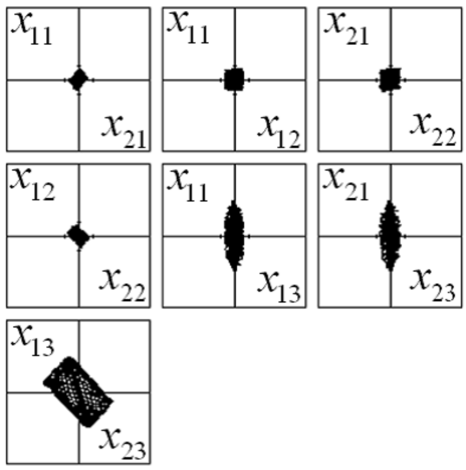

Fig. 3. Phase differences of six resonators. $\alpha_{C1} = 0.5$, $\alpha_{L1} = 0.3$, $\alpha_{C2} = 0.4$, $\alpha_{L2} = 0.2$, $\gamma = 0.01$, $\varepsilon = 0.055$ and $\beta = 3.14$.

Figure 2 shows the computer simulated results. The coupled oscillators exhibit various synchronization patterns for different initial conditions. Figure 3 shows the phase differences of voltages x_{11}-x_{13} and x_{21}-x_{23}. We can see from those figures that each phase difference is asynchronous. Hence, we can say that double-mode and simultaneous oscillation is generated at the same time. Namely, the relationship between right and left oscillators is double-mode, while the relationship between above and below oscillators is asynchronous. Figure 4 shows the time waveforms of the six voltages. The time waveforms of x_{11} and x_{21} are double-mode, while x_{12} and x_{22}, x_{13} and x_{23} are also double-mode. We can confirm that the envelopes of the double-mode oscillations are synchronized in anti-phase.

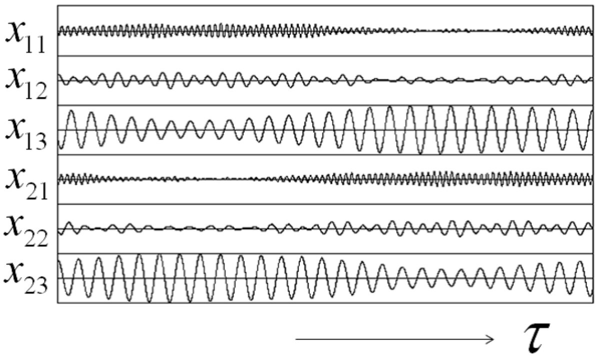

Fig. 4. Time waveforms of x_{mn}. $\alpha_{C1} = 0.5$, $\alpha_{L1} = 0.3$, $\alpha_{C2} = 0.4$, $\alpha_{L2} = 0.2$, $\gamma = 0.01$, $\varepsilon = 0.055$ and $\beta = 3.14$.

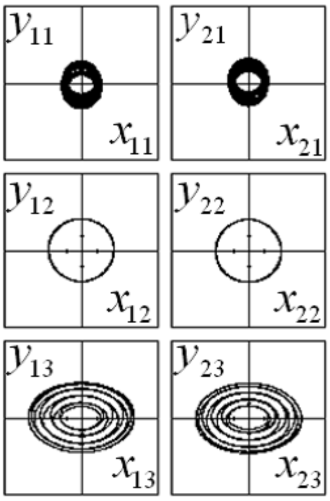

Fig. 5. Lissajous figures of six resonators. Horizontal: x_{mn}. Vertical: y_{mn}. $\alpha_{C1} = 0.5$, $\alpha_{L1} = 0.3$, $\alpha_{C2} = 0.4$, $\alpha_{L2} = 0.2$, $\gamma = 0.1$, $\varepsilon = 0.055$ and $\beta = 3.14$.

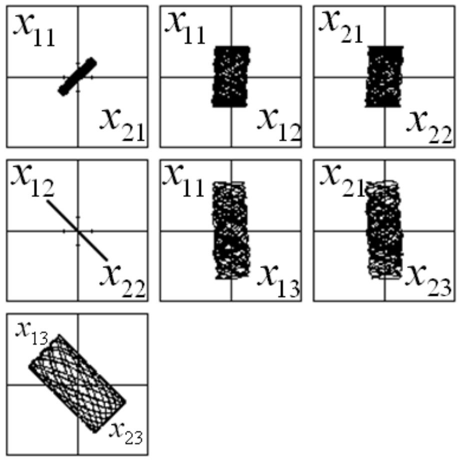

Fig. 6. Phase differences of six resonators. $\alpha_{C1} = 0.5$, $\alpha_{L1} = 0.3$, $\alpha_{C2} = 0.4$, $\alpha_{L2} = 0.2$, $\gamma = 0.1$, $\varepsilon = 0.055$ and $\beta = 3.14$.

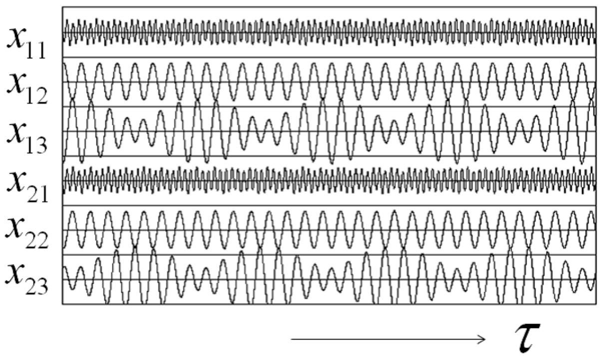

Fig. 7. Time waveforms of x_{mn}. $\alpha_{C1} = 0.5$, $\alpha_{L1} = 0.3$, $\alpha_{C2} = 0.4$, $\alpha_{L2} = 0.2$, $\gamma = 0.1$, $\varepsilon = 0.055$ and $\beta = 3.14$.

Figures 5, 6, and 7 show another example of the steady states obtained for different coupling parameter. The figure of x_{11}-x_{21} is synchronized in in-phase. But, the figure of x_{12}-x_{21} is synchronized in anti-phase, and the figure of x_{13}-x_{23} is double-mode.

IV. Conclusions

In this study, we have investigated the generation of synchronization phenomena observed from two inductively coupled simultaneous oscillators with three resonators. We could confirm the phenomenon that double-mode and simultaneous oscillation have generated at the same time. Namely, the relationship between right and left oscillators is double-mode, while the relationship between above and below oscillators is asynchronous.

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