

Synchronization of Coupled Oscillators with Double Tetrahedrons as a Shared Triangular Face

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Abstract— This paper acquaints the synchronization of coupled oscillatory system with double tetrahedral forms as a sharing triangular face. We confirm the synchronization phenomena by measuring the phase differences between adjacent oscillators. Also, we obtain the amplitude of each oscillator using the simulation results.

I. INTRODUCTION

Coupled oscillatory system is one of the most proper model to indicate the high-dimensional phenomena in natural science fields [1]-[7]. there are a lot of synchronization phenomena in natural environment. Therefore investigations of the coupled oscillatory systems about synchronization phenomena are reported in biology, physics and mathematics. Coupled oscillatory systems have various wave patterns and including wave propagations, clustering and complex patterns. Hence, various coupled oscillators were proposed, and the part of the mechanism of the non-linear phenomenon has been elucidated until now. However investigations on the complex coupled oscillatory networks have not been studied enough yet.

In our investigations, we use several van der Pol oscillators. Van der Pol oscillators have been coupled in various form and investigated about their synchronization phenomena [8], [9]. In particular, three coupled oscillatory system with a ring topology as shown Fig. 1 have made the very interesting results [10]. In this circuit system, each oscillator was coupled by an inductor and the number of coupled oscillators was an odd number. Then, we could not observe the synchronization phenomena with in/anti-phase states. That is to say, three-phase synchronization (phase shift: 120°) is obtained for the

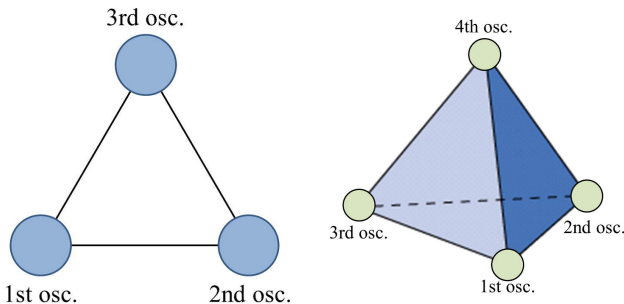


Fig. 1. Coupled van der Pol oscillators. (a) Three coupled oscillator. (b) Four coupled oscillator.

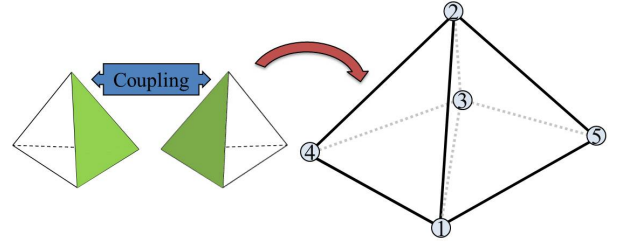


Fig. 2. Conceptual circuit model for tetrahedron form.

case of three oscillators by the effect of frustration. However, the three-phase synchronization was always observed stably in that system.

In our previous study, we have investigated several kinds of interesting synchronization phenomena in coupled oscillatory system which has stronger frustrations. We have researched four coupled van der Pol oscillators in the regular tetrahedron form as shown in Fig. 1(b). By computer simulation, we observed that the phase difference between adjacent oscillators changed and the synchronization was destroyed after the adjacent oscillators synchronize with anti-phase. And the another study, synchronization phenomena in two coupled triangular oscillatory networks sharing a branch was investigated in [11]. In this case, we could observed synchronization that the phase difference of sharing branch is in-phase (phase difference: 0°), and the other one between adjacent oscillators are synchronized with anti-phase (phase difference: 180°).

This paper presents synchronization of coupled oscillators with double tetrahedrons as a shared triangular face. We show the our coupled oscillatory system in Fig. 2 which is used tetrahedral forms sharing a triangular face. We confirm the synchronization phenomena by measuring the phase differences between adjacent oscillators.

II. CIRCUIT MODEL

The circuit model is shown in Fig. 3(a). In this circuit model, two tetrahedrons oscillators are coupled by the triangular face and the fourth and the fifth oscillators have no connection. In the computer simulations, we assume that the $v_k - i_{Rk}$ characteristics of nonlinear resistor in each oscillator is given by the following third order polynomial equation.

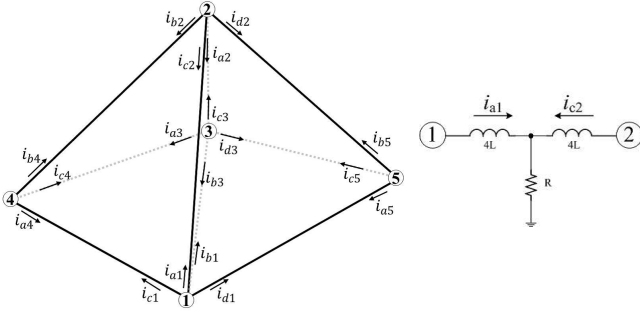


Fig. 3. Circuit model with double tetrahedrons. (a) Conceptual circuit model. (b) Coupled structure.

$$i_{Rk} = -g_1 v_k + g_3 v_k^3 \quad (g_1, g_3 > 0), \quad (k = 1, 2, 3, 4, 5). \quad (1)$$

The normalized circuit equations are expressed as:

$$\begin{cases} \frac{dx_k}{d\tau} = \varepsilon(1 - x_k^2)x_k - (y_{ak} + y_{bk} + y_{ck} + y_{dk}) \\ \frac{dy_{ak}}{d\tau} = \frac{1}{4}\{x_k - \eta y_{ak} - \gamma(y_{ak} + y_n)\} \\ \frac{dy_{bk}}{d\tau} = \frac{1}{4}\{x_k - \eta y_{bk} - \gamma(y_{bk} + y_n)\} \\ \frac{dy_{ck}}{d\tau} = \frac{1}{4}\{x_k - \eta y_{ck} - \gamma(y_{ck} + y_n)\} \\ \frac{dy_{dk}}{d\tau} = \frac{1}{4}\{x_k - \eta y_{dk} - \gamma(y_{dk} + y_n)\}. \end{cases} \quad (2)$$

We use the following normalizations:

$$\begin{aligned} t &= \sqrt{LC}\tau, \quad v_k = \sqrt{\frac{g_1}{g_3}} x_k, \quad i_{ak} = \sqrt{\frac{g_1 C}{g_3 L}} y_{ak}, \\ i_{bk} &= \sqrt{\frac{g_1 C}{g_3 L}} y_{bk}, \quad i_{ck} = \sqrt{\frac{g_1 C}{g_3 L}} y_{ck}, \\ \varepsilon &= g_1 \sqrt{\frac{L}{C}}, \quad \gamma = R \sqrt{\frac{C}{L}}, \quad \eta = r_m \sqrt{\frac{C}{L}}, \end{aligned} \quad (k = 1, 2, 3, 4, 5),$$

where ε is the nonlinearity, γ is the coupling strength, η indicates the resistive component and y_n denotes the current of neighbor oscillator on coupling resistor. In the computer simulations, we calculate the phase differences between adjacent oscillators. Here r_m denotes the internal resistance of an inductor.

III. SYNCHRONIZATION PHENOMENA

We calculate Eq. (2)-(6) using the fourth-order Runge-Kutta method with the step size $h = 0.002$. In this simulation, we consider frequency errors ζ at the voltage equation. In other word, the voltage equations of the second, the third, the forth and the fifth oscillators are multiplied by ζ ($k = 2, 3, 4, 5$),

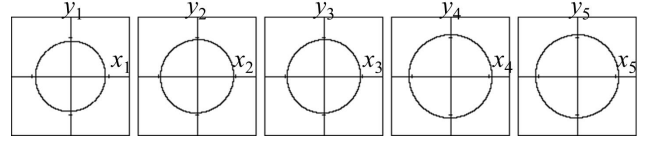


Fig. 4. Attractor between adjacent oscillators (horizontal axis: x_k , vertical axis: y_k ($k = 1, 2, 3, 4, 5$)).

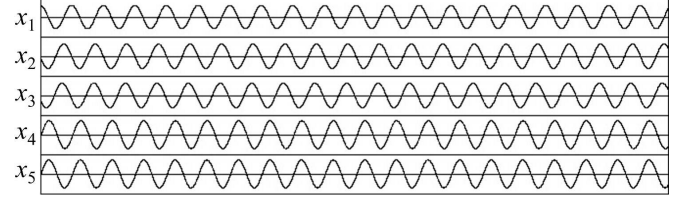


Fig. 5. The time waveform of the each oscillator.

we fix the frequency errors $\zeta_2 = 1.001$, $\zeta_3 = 1.002$, $\zeta_4 = 1.003$, $\zeta_5 = 1.004$.

We show the simulation result of the synchronization phenomena in Fig. 4. In this figure, we show the attractor of each oscillator and the horizontal axis is the voltage of each oscillator, and the vertical axis is the electric current of each oscillator. These electric currents are summed the four currents $y_k = y_{ak} + y_{bk} + y_{ck} + y_{dk}$ and we set the parameters $\varepsilon = 0.10$, $\gamma = 0.10$ and $\eta = 0.00010$. Because we define η to evade L -loop, we fix this value in these simulations. Next, the time waveforms of the voltage of each capacitor after sufficient time has elapsed are shown in Fig. 5. And the phase differences between the adjacent oscillators of this case is equal to the result. As a result, in the case of this circuit model, it was observed that the phase difference finally converged with the constant value. Also, this phase difference hardly varies even if we change the initial conditions. The case of parameter $\gamma > 0.0660$, the synchronization phenomena between each oscillator can be seen asynchronous.

When the coupling strength is strong, the phase differences converge for in/anti-phase. Shared triangular oscillators are synchronized for in-phase but the shared oscillators are

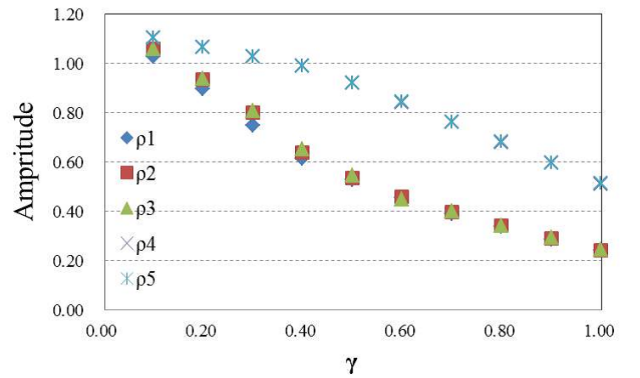


Fig. 6. Amplitude of voltage.

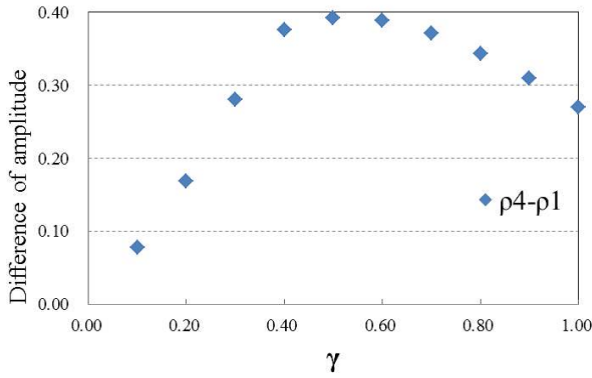


Fig. 7. Difference of amplitude.

synchronized with other oscillators for anti-phase. Also, We describe changes of the amplitude ρ_k of voltage in Fig. 6. In this result, the amplitudes are divided into two groups that one of the groups is the shared oscillators and the other one is the remained oscillators. We can find that all of amplitudes decrease by varying the parameter γ . Figure 7 shows the difference of amplitude $\rho_4 - \rho_1$, if the difference of the amplitude becomes big to some extent, each amplitude influences on the phase difference and each oscillator tends to converge for in/anti-phase.

IV. CONCLUSIONS

This paper has acquainted the synchronization of coupled oscillatory system with double tetrahedral forms as a sharing triangular face. We have confirmed the synchronization phenomena by measuring the phase differences between adjacent oscillators. Unlike the case only for the tetrahedron form, it was observed that the phase difference converged with certain values. Also, the coupling strength grew big, we could find that the amplitude of each oscillator decreased by using the simulation results.

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