Effective Associative Memory by Small World HNN with Adjusted Shortcut Connections

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1. Introduction

Associative memory is one of the important applications of the Hopfield Neural Network (HNN). This network model usually has full and symmetric connectivity. However, biological neural systems have local and non-symmetric connectivity. Also, the HNN causes high cost to generate the network in terms of implementation. A few years ago, Bohland et al. have proposed Small World HNN (SWHNN) [1] model in order to resolve these problems. The SW network improves ability for transmission by introducing the shortcut connections into the sparse regular network. We focus on the relationship between the SWHNN and the shortcut connections. In this study, in order to confirm the relationship between the SWHNN and the shortcut connections, we propose the SWHNN with Adjusted Shortcut Connections (ASC-SWHNN).

2. HNN working as Associative Memory

We consider the associative memory by the HNN. The HNN stores patterns by determining the weight parameter. The weight parameters \( w_{ij} \) is given by,

\[
    w_{ij} = \begin{cases} 
    \frac{1}{P} \sum_{p=1}^{P} x_i^{(p)} x_j^{(p)} & (i \neq j) \\ 
    0 & (i = j), 
    \end{cases} 
\]

where \( P \) is the number of patterns and \( x \) is the pattern of memory. All neurons are initialized by an unknown pattern. Namely, the network is given the input. The state of each neuron is determined by Eq. (2).

\[
    u_i(t) = \sum_{j=1}^{n} w_{ij}(t)x_j(t), \quad (2)
\]

where \( u \) is the internal state of the neuron and \( x \) is the input or output. The value of \( u \) is binary. Neuron’s output is determined by a threshold function. The equation of the determining neuron’s output is given by Eq. (3).

\[
    x_i(t+1) = \begin{cases} 
    1, & (u_i(t) \geq 0) \\ 
    -1, & (u_i(t) < 0). 
    \end{cases} \quad (3)
\]

The HNN recalls a similar pattern to the input pattern from the stored patterns by repeating Eq. (2) to Eq. (3). The SWHNN is the SW network which is trained using these normal HNN rule.

3. ASC-SWHNN Model

First, we consider a one-dimensional ring model of the SW network model. We start with a \( n \) ring regular lattice, with each unit connected to its \( k \) nearest neighbors by edges, and then rewire with probability \( p \). The one-dimensional ring model is shown in Fig. 1. The ASC-SWHNN is composed based on the SW network and is adjusted by multiplying the weight parameters of the shortcut connections in the SWHNN by an adjusting coefficient \( \alpha \). Here, the shortcut connections are a few longer-range edges than edges which connects each unit to its \( k \) nearest neighbors.

Figure 1: Example of the one-dimensional ring model of the SW network, when \( n = 16 \) and \( k = 4 \).

4. Simulation Results

In this study, we apply three networks, which are the HNN based on the regular network \( \{1\} \), the SWHNN \( \{2\} \) and the ASC-SWHNN \( \{3\} \), to the associative memory. Also, we compare the recall rate of three networks when \( k \) is changed. In the computer simulations, we consider each network with 400 neurons. We prepare 3 stored binary patterns at random and produce \( 3 \times 50 \) binary patterns to be input, which differs from each stored patterns by 25%. It is practically reasonable that a pattern within 1% error is considered as being recalled. In the case of the SWHNN or the ASC-SWHNN, we randomly compose 10 network and calculate the average of the recall rate of the networks. The simulation results are shown in Fig. 2 and show that the ASC-SWHNN provides the better recall performance than other networks. Therefore, we assume that the propagation of the SW network improves by the strong shortcut connections.

Figure 2: Process of recall in different networks, the sparse HNN, the SWHNN and the ASC-SWHNN, when \( p = 0.1 \).

5. Conclusions

In this study, we have proposed the ASC-SWHNN and apply the associative memory. The computer simulation results showed that the ASC-SWHNN provided the better recall performance than other networks. We assumed that the propagation of the SW network improves by the strong shortcut connections.

References