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Clustering Phenomenon in Two-Dimensional Lattice of Logistic Maps

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1. Introduction

Chaos is the phenomenon that produces completely different results by small differences in initial conditions. Logistic Map is one of the most famous chaos-generating function. This function can be defined as a very simple formula as follows:

$$x_{n+1} = ax_n(1 - x_n) \tag{1}$$

By changing the initial value x_0 or the parameter a of Eq. (1) slightly, the equation gives significantly different results.

In this study, we focus on the two-dimensional lattice of the Logistic Map, because the one-dimensional ladder has been studied by many engineers [1]. We investigate complex behavior by varying the parameter of the Logistic map and the strength of coupling.

2. System model

Figure 1 shows the structure of the two-dimensional lattice of the Logistic Map. We consider the ring boundary conditions.

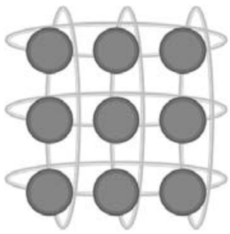


Figure 1: Two-dimensional lattice.

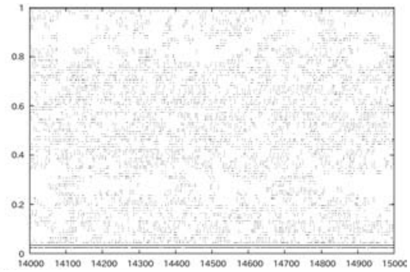
The equations describing the system are defined as follows:

$$\begin{aligned} x[i][j] \quad = \quad & (1-\varepsilon)\, a\, x[i][j]\, (1.0 - x[i][j]) \\ & + 0.25\, \varepsilon\, (4x[i][j] - x[i][j+1] - \\ & x[i][j-1] - x[i-1][j] - x[i+1][j]) \\ & (i, j = 1, 2, 3, \cdots, N) \end{aligned} \tag{2}$$

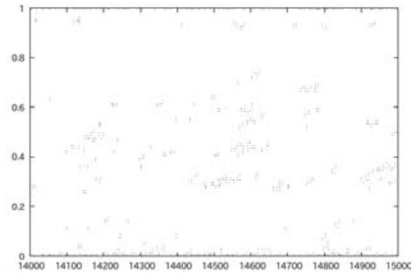
3. Simulation results

In this article, we fix the number of the Logistic map as N=100, namely 100× 100 lattice is considered. We also fix $a = 3.83$ and vary the strength of the coupling ε.

We focus on the difference between the adjacent Logistic Maps in order to investigate the clustering patterns. Figure 2 shows some examples of the space-time diagrams obtained by computer simulations. In the diagram, we extracted the data only on the first row ($i = 1$) because of the huge quantities of data for the whole system. The horizontal axis is iteration (time) and the vertical axis is the information of synchronization (space). If $|x[1][j] - x[1][j+1]|$ is larger than 0.1, the corresponding space-time pixel is painted as black, while it is left blank otherwise. So, the white area can be regarded as a cluster.



(a)



(b)

Figure 2: Space-time diagram for 100×100 lattice. (a) $\varepsilon = 0.0530$. (b) $\varepsilon = 0.0555$.

For smaller coupling strength, many small clusters appear but disappear soon as shown in Fig. 2(a). On the other hand, for relatively large coupling strength, small number of large size clusters appear in Fig. 2(b). But, at the same time, this state can be also said as a mingled state of synchronous and asynchronous states.

We expect that this phenomenon will be observed if we extract another row or another column, because of the symmetry of the system. Namely, extremely complicated clustering phenomenon can be confirmed to occur in this two-dimensional lattice.

4. Conclusions

As a result, the clustering phenomenon was confirmed in the two-dimensional lattice of the Logistic Map. In the future work, we investigate the synchronous rate and also apply this phenomenon to the neural networks solving combinatorial optimization problems.

References

[1] K. Kaneko, "Supertransients, spatiotemporal intermittency and stability of fully developed spatiotemporal chaos," *Physics Letters A*, vol. 149, no. 2&3, pp. 105-112, 1990.