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SPICE-Oriented Analysis of Oscillator by Using Averaging Method

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1. Introduction

In the field of electrical engineering, a lot of studies on synchronization phenomena of coupled van der Pol oscillators have been carried out up to now. In generally, averaging method [1] is widely used for analysis of weakly nonlinear systems in stable state. However, the number of variables and equilibrium points will be too large, by increasing the number of coupled oscillators. In this case, it is not easy to find all of equilibrium points.

In this study, we propose a convenient algorithm of averaging method combined with Newton homotopy method by using SPICE. By combining averaging method and Newton homotopy method, we obtain multiple equilibrium points with a single SPICE simulation in parallel.

2. Proposal method

Averaging method is used for solving the weakly nonlinear circuit systems.

$$\dot{x} = y,\ \dot{y} = \varepsilon f(t, x, \dot{x}) - x. \tag{1}$$

In Eq. (1), $f(t, x, \dot{x})$ is nonlinear function and ε denotes the tiny constant. Namely,

$$x = \rho(t)\sin(t + \theta(t)),\ y = \rho(t)\cos(t + \theta(t)). \tag{2}$$

All of oscillatory circuits can be expressed by Eq. (3)(is obtained by Eq. (1) and Eq. (2)).

$$\begin{cases} 0 = \dfrac{\varepsilon}{2\pi}\displaystyle\int_0^{2\pi} f(\phi-\theta, \rho\sin\phi, \rho\cos\phi)\cdot\cos\phi d\phi \\ 0 = -\dfrac{\varepsilon}{2\pi\rho}\displaystyle\int_0^{2\pi} f(\phi-\theta, \rho\sin\phi, \rho\cos\phi)\cdot\sin\phi d\phi \\ \phi \equiv t + \theta \end{cases}. \tag{3}$$

It is obtained by

We combine Newton homotopy method to Eq. (3). Namely,

$$\begin{cases} 0 = \dfrac{\varepsilon}{2\pi}\displaystyle\int_0^{2\pi} f(\phi-\theta, \rho\sin\phi, \rho\cos\phi)\cdot\cos\phi d\phi \\ \quad -(1-\tau)f_0 \\ 0 = -\dfrac{\varepsilon}{2\pi\rho}\displaystyle\int_0^{2\pi} f(\phi-\theta, \rho\sin\phi, \rho\cos\phi)\cdot\sin\phi d\phi \\ \quad -(1-\tau)f_1 \\ \left(\dfrac{d\rho}{ds}\right)^2 + \left(\dfrac{d\phi}{ds}\right)^2 + \left(\dfrac{d\tau}{ds}\right)^2 = 1 \end{cases}. \tag{4}$$

In Eq. (4), initial state is set by a points f_1 and f_2, and gets the solutions at $\tau = 1$ on the path. We make the circuit model satisfies Eq. (4). $f(\theta, \rho, \phi)$ is realized by analog behavior models (ABMs) in SPICE. ABM is the SPICE-oriented function for realize an equation.

3. Illustrative example and result

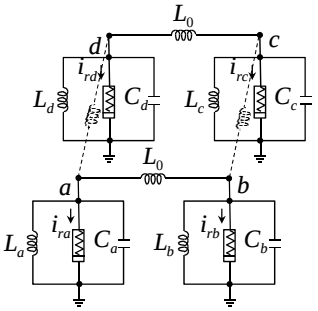

Figure 1: Example model.

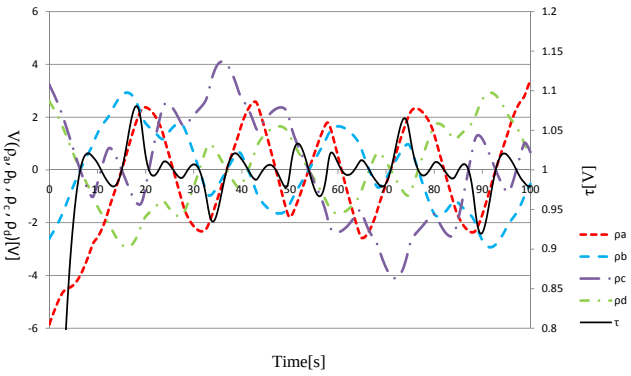

Figure 2: Simulation result.

As an example, we analyze four pan der Pol coupled by inductor as Fig. 1. Figure 2 shows simulation result which obtained by transient analysis of SPICE.

We found sixteen equilibrium points from Fig. 2. Equilibrium points are asymptotic stability, when real part of eigenvalue satisfy < 0. They includes five stable points (time $= 6.40, 15.03, 36.86, 70.96, 92.78$). It is calculated by partial differential equation of Eq. (3).

4. Conclusions

We proposed convenient algorithm of averaging method combined with Newton homotopy method by using SPICE. We found multiple equilibrium points with a single SPICE simulation. As a future work, we would like to apply the proposed method to another patterns of connection for oscillators.

References

[1] T. Endo and S. Mori, "*Mode Analysis of Two -Dimensional Low-Pass Multimode Oscillator,*" *IEEE Trans. Circuits and Syst., vol. 23, pp. 517-530, Sep. 1976.*