Ant Colony Optimization Changing the Rate of Dull Ants for QAP

Sho SHIMOMURA$^1$, Haruna MATSUSHITA$^2$ and Yoshifumi NISHIO$^1$
(Tokushima University$^1$, Kagawa University$^2$)

1. Introduction

Ant Colony Optimization (ACO) is a biologically inspired optimization algorithm with pheromone effects of ants and is effective to solve difficult combinatorial optimization problems like the Quadratic Assignment Problem (QAP). QAP is one of the NP-hard combinatorial optimization problems.

In our previous study, we have proposed a new ACO algorithm; ACO with Intelligent and Dull ants (IDACO) [1]. In IDACO, two kinds of ants coexist: intelligent ant and dull ant. The intelligent ant can trail the pheromone, however the dull ants cannot trail the pheromone. We have confirmed that IDACO obtained effective result for the Traveling Salesman Problems.

In this study, we propose an improved IDACO; ACO Changing the Rate of Dull Ants (IDACO-CR).

In addition to the existence of the dull ants, the rate of dull ants in IDACO-CR are flexibly changes in the optimization. We apply IDACO-CR to QAPs.

2. IDACO-CR

Given two matrices, a distance matrix $D$ and a flow matrix $F$, find a permutation $\Pi$ which corresponds to the minimum value of the total assignment cost $L$ in Eq. (1).

\[
L = \sum_{i=1}^{n} \sum_{j=1}^{n} D_{ij} F_{\pi(i)\pi(j)},
\]

where $D_{ij}$ and $F_{ij}$ are the $(i,j)$-th elements of $D$ and $F$, respectively, $\pi(i)$ is the $i$-th element of the vector $\Pi$, and $n$ is the size of the problem. The number of ants is denoted by $M$. $d \times M$ ants are classified into a set of the dull ants $S_{\text{dull}}$. $d$ is a rate of dull ants in all the ants.

[Step1] Let $t = 0$. $\tau_{ij}(t)$ is the amount of pheromone trail on a coupling $(i, j)$ to assign the activity $j$ to the location $i$, and $\tau_{ij}(0) = 10$.

[Step2] From the two vectors $D$ and $F$, one may obtain a matrix $A$. The element $a_{ij}$ of $A$ is given by the product $d_{ij} \cdot f_{ij}$. IDACO-CR uses the two kinds of choice probabilities. The probability that $k$-th ant $(k = 1, \cdots, M)$ assigns activity $j$ to location $i$ is decided by

\[
p_{kj}(t) = \begin{cases} \frac{[\eta_{ij}]^{\beta_D}}{\sum_{l \in N^k} [\eta_{il}]^{\beta_D}}, & \text{if } k \in S_{\text{dull}} \\ \frac{[\tau_{ij}(t)]^{\alpha} [\eta_{ij}]^{\beta_D}}{\sum_{l \in N^k} [\tau_{il}(t)]^{\alpha} [\eta_{il}]^{\beta_D}}, & \text{otherwise,} \end{cases}
\]

where $1/\eta_{ij}$ is the element $a_{ij}$. The adjustable parameters $\alpha$, $\beta$ and $\beta_D$ control the weight of pheromone intensity and of the element, respectively. $N^k$ is a set of activities such that $k$-th ant has not yet assigned any activity in the set.

[Step3] After all the ants have completed assignment, compute the total cost $L_k(t)$ and update the amount of the pheromone $\tau_{ij}(t)$. Note that dull ants can deposit the pheromone, though, they cannot trail the pheromone. The amount of the pheromone $\Delta\tau_{ij}^k$ deposited by $k$-th ant on the coupling $(i,j)$ is decided as

\[
\Delta\tau_{ij}^k(t) = \begin{cases} 10/L^k, & \text{if } (i,j) \in T^k(t) \\ 0, & \text{otherwise,} \end{cases}
\]

where $T^k(t)$ is obtained permutation by $k$-th ant and $L_k(t)$ is its total cost. $\rho \in [0,1]$ is the pheromone trail decay coefficient. Update $\tau_{ij}(t)$ of each coupling $(i,j)$ depending on its $\Delta\tau_{ij}^k$:

\[
\tau_{ij}(t+1) = \rho \tau_{ij}(t) + \sum_{k=1}^{M} \Delta\tau_{ij}^k(t).
\]

[Step4] The rate of dull ants $d$ is changed if the system is trapped into the local optima. If the best solution is not updated within 2000 times, IDACO-CR is judged trapped into local solution. The rate of dull ants $d$ is changed according to

\[
d = \begin{cases} d + d_\text{up} & \text{if trapped local solution} \\ 0, & \text{otherwise,} \end{cases}
\]

where the adjustable parameter $d_\text{up}$ controls the addition of the rate of dull ants. Thus, the rate of dull ants $d$ is flexibly varied from 0 to 1.0. If the best solution is updated, $d$ is reset to the initial value $d = 0$.

[Step5] Let $t = t + 1$. Go back to [Step2] and repeat until the maximum time limit $t = t_{\text{max}}$.

3. Numerical Experiments

We apply IDACO-CR to three QAPs and compare IDACO-CR with the standard ACO and the conventional IDACO. The rate of dull ants $d$ in the conventional IDACO is fixed. In the experiments, the number of ants $M$ is set to the same as the number of locations. We repeat the simulation 20 times for all the problems. The parameters are set to the follows; $\rho = 0.9$, $\alpha = 1$, $\beta = \beta_D = 1$, $t_{\text{max}} = 15000$, $d_\text{up} = 0.2$.

The simulation results are shown in Table 1. We can confirm that IDACO-CR obtains better results than the other methods. We can see that changing the rate of dull ants in the simulation is more effective than using the fixed rate of dull ants.

Table 1: Results of IDACO-CR, the standard ACO and the conventional IDACO with $d = 0.5$.

<table>
<thead>
<tr>
<th></th>
<th>Nug22</th>
<th>Scr12</th>
<th>Had20</th>
</tr>
</thead>
<tbody>
<tr>
<td>The standard ACO</td>
<td>4.76%</td>
<td>7.95%</td>
<td>3.83%</td>
</tr>
<tr>
<td>IDACO with $d = 0.5$</td>
<td>5.88%</td>
<td>8.16%</td>
<td>3.91%</td>
</tr>
<tr>
<td>IDACO-CR</td>
<td>4.57%</td>
<td>7.83%</td>
<td>3.62%</td>
</tr>
</tbody>
</table>

4. Conclusions

In this study, we have applied IDACO-CR to QAPs. We have confirmed that IDACO-CR obtained better results than the standard ACO and the conventional IDACO because IDACO-CR can change the rate of dull ants depending on the problems.

References