

# The Hybrid of Ant Colony Optimization and Simulated Annealing

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## 1. Introduction

Ant Colony Optimization (ACO) is an optimization algorithm imitating ant's feeding action and is effective to solve combinatorial optimization problems like the Traveling Salesman Problems (TSPs)[1]. It is well known that TSP can be solved by using various kinds of Algorithms such as Genetic Algorithm (GA), Simulated Annealing (SA) and so on. However it is difficult to find the optimal solution by increasing the number of elements.

In this study, we propose the hybrid of Ant Colony Optimization and Simulated Annealing (ACOSA). ACOSA has three important features. First, ACO and SA are operated in parallel. Second, the information obtained from SA is treated as a honey which is different with a pheromone. Third, the honey affects the searching solutions of ACO. We apply ACOSA to two TSPs and confirm its effectiveness.

## 2. ACOSA

In ACOSA algorithm, ACO and SA make tour, respectively. The tour made by SA is similarly estimated as a pheromone and affects the search probability of ACO. The evaluation value of the tour made by SA is called honey.

**[ACOSA 1]**(Initialization): Let iteration number  $t = 0$ .  $\tau_{ij}(t)$  is the amount of pheromone trail on the path ( $ij$ ) between cities  $i$  and  $j$  at time  $t$ , and  $\tau_{ij}(t)$  is initially set to  $\tau_0$ .  $m_{ij}(t)$  is the amount of honey trail on the path ( $ij$ ) between cities  $i$  and  $j$  at time  $t$ , and  $m_{ij}(t)$  is initially set to  $m_0$ .

**[ACOSA 2]**(Find tour):

· Find tour of ACO: For the  $k$ -th ant ( $k = 1, 2, \dots, M$ ), the visiting city is chosen by probability  $p_{ij}^k(t)$ . ACOSA uses two kinds of  $p_{ij}^k(t)$  according to conditions. The  $k$ -th ant finds a tour according to the following equations;

$$p_{ij}^k(t) = \begin{cases} \frac{[\tau_{ij}]^\alpha [\eta_{ij}]^\beta [m_{ij}]}{\sum_{h=\Omega} [\tau_{ih}]^\alpha [\eta_{ih}]^\beta [m_{ih}]}, & \text{if } L_S \geq L_A \\ \frac{[\eta_{ij}]^\beta [m_{ij}]^\gamma}{\sum_{h=\Omega} [\eta_{ih}]^\beta [m_{ih}]^\gamma}, & \text{otherwise,} \end{cases} \quad (1)$$

where  $L_A$  is the best tour length of ants at  $t$ ,  $L_S$  is the best tour length of SA at  $t$ .  $\Omega$  is a city set which ants are not visiting yet.  $\alpha$ ,  $\beta$ , and  $\gamma$  are the control parameters of  $\tau$ ,  $\eta$ , and  $m$ , respectively.

· Find tour of SA:

1: City is set at random and tour is created. This tour length is the present state  $s$ .

2: Two of the city turn of  $s$  are exchanged and tour is created. This is performed for the number of cities, and minimum length in its tours is the neighborhood state  $e$ .

3: Evaluation and updating are processed as follows.

Finds a tour according to the following equations;

$$\text{if } s_b \geq s(t) \quad s_b = s(t), \quad (2)$$

$$s(t+1) = \begin{cases} e, & s(t) \geq e \\ e, & s(t) < e: \text{ follow the prob. } p_s(t) \\ s(t), & s(t) < e: \text{ follow the prob. } p_s(t) \end{cases} \quad (3)$$

where  $s_b$  is the best state.

$$p_s(t) = \exp \frac{s(t) - e}{T}, \quad (4)$$

where  $T$  is a temperature parameter.

**[ACOSA 3]**(Pheromone update): Compute the tour length  $L_k(t)$  and update the amount of the pheromone trail  $\tau_{ij}$  by

$$\tau_{ij}(t+1) = (1 - \rho)\tau_{ij}(t) + \sum_{k=1}^M \Delta\tau_{ij}^k(t), \quad (5)$$

$$\Delta\tau_{ij}^k(t) = \begin{cases} 1/L_k, & \text{if tour made by ant} \\ 0, & \text{otherwise,} \end{cases} \quad (6)$$

where  $\rho \in [0, 1]$  is the pheromone trail decay coefficient.

**[ACOSA 4]**(Honey update): Compute the tour length  $s_b$  and update the amount of the honey by

$$m_{ij}(t+1) = 1/s_b \quad (7)$$

**[ACOSA 5]** Let  $t = t + 1$  and  $T * c$ .  $c \in [0, 1]$  is cooling coefficient and makes  $T$  low gradually. Go to [ACOSA 2] and repeat until the maximum time limit  $t = t_{max}$ .

## 3. Numerical Experiments

In order to confirm the effectiveness of ACOSA, we apply ACOSA to two TSPs, ulysses16 and att48. The total number of the ants of ACO and ACOSA are set to the same the number of cities. We carry out the simulation 2000 iterations for 10 times. The simulation results of ACO and ACOSA are shown in Table 1. In this table, we can confirm that ACOSA obtains better result than standard ACO in both of TSPs. From this results, we consider that honey has a good effect on search solution of ACO. SA performs changing solution for the worse (adoption of a long tour) in an early stage. We consider that this algorithm diversify search solution of ACO.

Table 1: Results of ACO and ACOSA for ulysses16 and att48.

		ulysses16	att48
ACO	AVG	0.89 %	2.52 %
	MIN	0.15 %	2.30 %
ACOSA	AVG	<b>0.10 %</b>	<b>2.19 %</b>
	MIN	0 %	0.80 %

$$\begin{cases} \text{AVG} = (\text{Avg.tour} - \text{Optimal tour}) / \text{Optimal tour} * 100 \\ \text{MIN} = (\text{Min.tour} - \text{Optimal tour}) / \text{Optimal tour} * 100 \end{cases}$$

## 4. Conclusions

In this study, we have proposed the hybrid of Ant Colony Optimization and Simulated Annealing (ACOSA). ACOSA obtained the better results than standard ACO. In particular, the minimum has been improved greatly because SA diversify search solution of ACO.

## Reference

[1] M. Dorigo and L. M. Gambardella, "Ant Colonies for the Traveling Salesman Problem," *BioSystems*, vol. 43, pp. 73-81, 1997.