1-2

Synchronization Phenomena in Cross-coupled Two Colpitts Oscillators

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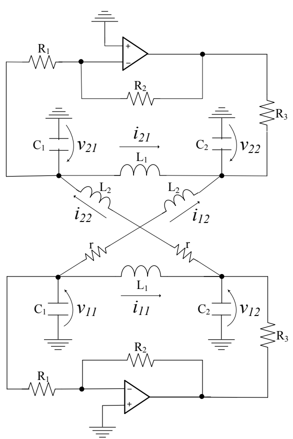
1. Introduction

Synchronization phenomena are observed everywhere in natures, such as vibration of a pendulum, the luminescence phenomena of a firefly, and so on. Also, synchronization with electric circuit is effective in analyzing the natural phenomena. It is easy to observe the synchronization phenomena of coupled oscillators by various elements and techniques.

In this study, we observe synchronization phenomena in cross-coupled two Colpitts oscillators by using computer simulations and circuit experiments.

2. Circuit Model

We show the our circuit model in Fig.1. In this study, we investigate the case where two Colpitts oscillators are cross-coupled via inductors.



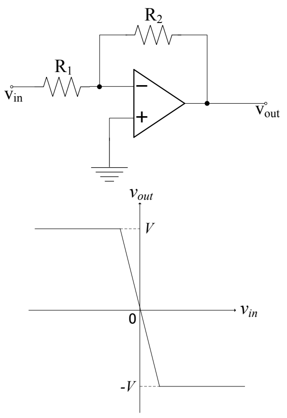


Fig 1: Circuit model. Fig 2: OP-Amp model.

The operational amplifier (OP-Amp) currently used for our circuit model is not an ideal linear function, and we assume that the OP-Amp model is taken as the piecewise linear function as shown in Fig. 2.

Therefore, input-output characteristics of an OP-Amp is expressed with Eq. (1).

$$v_{out} = \frac{R_2}{2R_1}\left(|v_{in} - \frac{R_1}{R_2}V| - |v_{in} + \frac{R_1}{R_2}V|\right) \qquad (1)$$

The normalized circuit equations of our circuit model are given as follows:

$$\begin{cases} \dot{x}_{11} = -\alpha_1\, x_{11} - \dfrac{1}{\alpha_3}\rho\, y_{12} - \dfrac{1}{\alpha_3}\, y_{11} \\ \dot{x}_{12} = -\alpha_3\, \varepsilon\, x_{12} + \dfrac{\gamma}{2}\left(|x_{11} - \dfrac{1}{\gamma}| - |x_{11} + \dfrac{1}{\gamma}|\right) \\ \qquad\quad + \dfrac{1}{\alpha_3}\, y_{11} - \dfrac{1}{\alpha_3}\rho\, y_{22} \\ \dot{y}_{11} = \alpha_3\,(x_{11} - \varepsilon\, x_{12}) \\ \dot{y}_{12} = \alpha_3\,(x_{11} - \varepsilon\, x_{22}) - \dfrac{1}{\alpha_r}\rho\, y_{12} \end{cases} \qquad (2)$$

$$\begin{cases} \dot{x}_{21} = -\alpha_1\, x_{21} + \dfrac{1}{\alpha_3}\rho\, y_{22} - \dfrac{1}{\alpha_3}\, y_{21} \\ \dot{x}_{22} = -\alpha_3\, \varepsilon\, x_{22} + \dfrac{\gamma}{2}\left(|x_{21} - \dfrac{1}{\gamma}| - |x_{21} + \dfrac{1}{\gamma}|\right) \\ \qquad\quad + \dfrac{1}{\alpha_3}\, y_{21} + \dfrac{1}{\alpha_3}\rho\, y_{12} \\ \dot{y}_{21} = \alpha_3\,(x_{21} - \varepsilon\, x_{22}) \\ \dot{y}_{22} = \alpha_3\,(\varepsilon\, x_{12} - x_{21}) - \dfrac{1}{\alpha_r}\rho\, y_{22} \end{cases} \qquad (3)$$

We use the following normalizations:

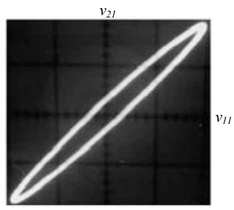
$$\gamma = (R_2/R_1R_3)\sqrt{L/C_1},$$

$$\alpha_r = (1/r)\sqrt{L/C_1},\ \alpha_1 = (1/R_1)\sqrt{L/C_1},$$

$$\alpha_3 = (1/R_3)\sqrt{L/C_1},\ \varepsilon = C_1/C_2,\ \rho = L_1/L_2.$$

3. Synchronization Phenomena

Figure 3 is the circuit experiment result, (horizontal : [10 V/div], vertical : [10 V/div], $C_1 = 1$ nF, $C_2 = 10$ nF, $R_1 = 10$ kΩ, $R_2 = 100$ kΩ, $R_3 = 1$ kΩ, $L_1 = 10$ mH, $L_2 = 5$ mH). Figure 4 is the computer simulation result by the normalized Eq. (2) and (3). Attractor on $\dot{x}_{11}$ - $\dot{x}_{21}$ plane, $\gamma = 31.62$, $\alpha_1 = 0.3162$, $\alpha_3 = 3.162$, $\alpha_r = 316.2$, $\varepsilon = 0.10$, $\rho = 2.0$.



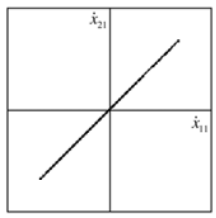


Fig 3: Experiment result. Fig 4: Simulation result.

We show the in-phase synchronization in Fig.2 and Fig.3. When we compare two Lissajous figures, phase difference of computer simulation result is 0.04°, but Lissajous figure of circuit experiment result is different. As the cause, we assume that two oscillators in our circuit experiment does not have a perfectly same elements.

Furthermore, we summarize synchronization phenomena in circuit experiment by change of L_2 in Table 1. We observe in-phase state when L_2 is less than 50mH, asynchronous state when L_2 is more than 50mH.

Tab. 1: Synchronization state by change of L_2.

L_2 [mH]	synchronization state
5〜50	in-phase
more than 50	asynchronous

4. Conclusions

In this study, we have observed synchronization phenomena in the proposed circuit. In the future, we would like to investigate what kind of phenomena it becomes about the case of four oscillators are coupled.