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In-Phase and Anti-Phase Synchronization of Switching Phenomena in Coupled Chaotic Circuits

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***Abstract*— Switching phenomena of attractors can be observed in coupled chaotic circuits. In this study, we confirmed that in-phase and anti-phase synchronizations of switching phenomena. The phenomena can be observed in asynchronous states. Therefore, it is very interesting phenomena.**

I. Introduction

Many kinds of complex phenomena are observed on large-scale coupled chaotic circuits. Investigations of these phenomena are very important works in order to declare nonlinear phenomena in the natural world. Electric circuits are suitable for models of large-scale coupled nonlinear systems. Because getting electric parts is easily, the price is inexpensive, experiment time is very short, repeatability of experiments is very high, an electric circuit is real physical system and so on. Therefore there are many studies of large-scale coupled circuit systems. In these studies, synchronization phenomena are attracted attentions.

On the other hand, some chaotic circuits have coexisting attractors. In these circuits, switching phenomena of attractors are observed by changing parameters. Normally, in the case of synchronization states, switching of attractors are also synchronized. And in the case of asynchrization states, switching of attractors are also asynchronized. However, there is the possibility that switching of attractors are synchronized in the case of asynchrization states.

In this study, we investigate coupled chaotic circuits and confirm in-phase and anti-phase synchronizations of switching phenomena. Especially, the case that the number of circuit is two is investigated.

II. System model

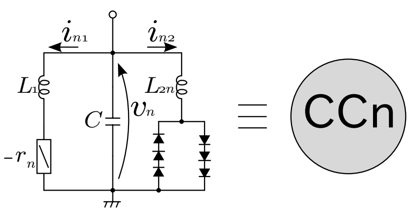

Fig. 1. Circuit model

A circuit model [1] using in this study is shown in Fig. 1. This circuit consists of three memory elements, one linear negative resistor and bi-directionally-coupled diodes. In this circuit, two attractors which are symmetrical about a origin are observed by adjusting the value of the negative resistor.

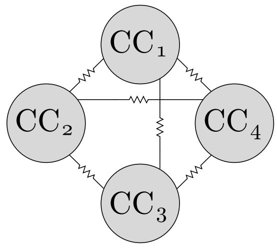

Fig. 2. System model ($N = 4$)

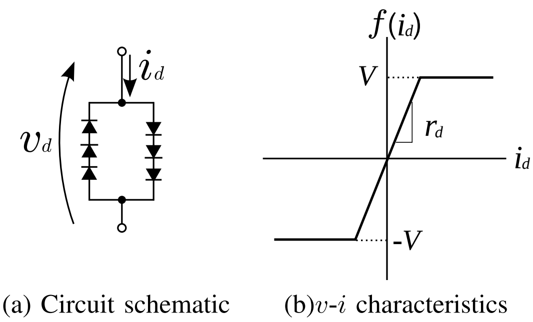

Fig. 3. Coupled diodes model

In this study, we investigate coupled chaotic circuits which is coupled by resistors shown in Fig. 2. Firstly, a system equation is derived. Bi-directionally-coupled diodes are modeled as Fig. 3. Then, we can derive a following function

$$v_d = \left(\left| i_d + \frac{V}{r_d} \right| - \left| i_d - \frac{V}{r_d} \right| \right). \tag{1}$$

The others elements are modeled as linear elements. Each circuit number is defined as $1 \le n \le N$. System equation is described as follows:

$$\left\{\begin{array}{rcl} L_1\dfrac{di_{n1}}{dt} &=& v_n + r_n i_{n1}, \\ L_{2n}\dfrac{di_{n2}}{dt} &=& v_n - \dfrac{r_d}{2}\left(\left|i_{n2}+\dfrac{V}{r_d}\right| - \left|i_{n2}-\dfrac{V}{r_d}\right|\right), \\ C\dfrac{dv_n}{dt} &=& -\left(i_{n1}+i_{n2}\right) - G\left(Nv_n - \displaystyle\sum_{k=1}^{N} v_k\right). \end{array}\right. \tag{2}$$

Changing parameters and variables as follows,

$$\begin{array}{l} t = \sqrt{L_1 C}\tau,\quad i_{n1} = V\sqrt{\dfrac{C}{L_1}}x_n,\quad i_{n2} = V\sqrt{\dfrac{C}{L_1}}y_n, \\ v_n = Vz_n,\quad \text{"}\cdot\text{"} = \dfrac{d}{d\tau},\quad \alpha_n = r_n\sqrt{\dfrac{C}{L_1}}, \\ \beta_n = \dfrac{L_1}{L_{2n}},\quad \gamma = \sqrt{\dfrac{C}{L_1}}r_d \text{ and } \delta = G\sqrt{\dfrac{L_1}{C}}. \end{array} \tag{3}$$

The normalized system equation is described as follows.

$$\left\{\begin{array}{l} \dot{x}_n = z_n + \alpha_n x_n, \\ \dot{y}_n = \beta_n\left\{z_n - \dfrac{\gamma}{2}\left(\left|y_n+\dfrac{1}{\gamma}\right| - \left|y_n-\dfrac{1}{\gamma}\right|\right)\right\}, \\ \dot{z}_n = -x_n - y_n - \delta\left(Nz_n - \displaystyle\sum_{k=1}^{N} z_k\right). \end{array}\right. \tag{4}$$

In the next section, by using this equation, computer simulations are carried out.

III. Computer Simulations

A. All phenomena observed in this study

Firstly, all phenomena observed in this study are shown. Figure 4 shows a coexisting attractors of a chaotic circuit as shown in Fig. 1. The orbit is color-coded according to two attractors. This color coding is defined by an investigation of a bifurcation diagram [2]. In following all simulation results, this definition is applied.

Figure 5 shows a computer simulation result in the case of an anti-phase synchronization of switching phenomena. Figure 5 (a) shows waveforms of two circuits. Horizontal axis show time. Vertical axes show z_1 and z_2. These are corresponding to voltage v_n of the circuit model as shown in Fig. 1. Figure 5 (b) shows a difference of two waveforms. Horizontal axes show time. Vertical axis shows as follows. A black line is z_1 - z_2. A green line shows a difference of two attractors. Namely, when both of z_1 and z_2 is red or blue, the line becomes 0. When z_1 is blue and z_2 is red, the line becomes 2. When z_1 is red and z_2 is blue, the line becomes -2. This rule is applied to following simulation results. In this case, z_1 and z_2 is not synchronized and switching phenomena of two attractors are synchronized with anti-phase. Figure 6 shows a computer simulation result in the case of in-phase synchronization of switching phenomena. In this case, z_1 and z_2 are not synchronized and two attractors are synchronized with in-phase.

Above two kinds of phenomena are observed in a steady state. We consider that these are interesting phenomena.

Figure 7 shows a computer simulation result in the case of an asynchronization of switching phenomena. Figure 8 shows a computer simulation result in the case of switching phenomena on one circuit only. In this case, if circuits are not coupled, z_2 keeps red in a steady state. Figure 9 shows a computer simulation result in the case of holding one attractor state in each circuit. In this case, by changing initial values, an opposite pattern of the Fig. 9 is also observed. Figure 10 shows a computer simulation result in the case of boundary area between Fig. 5 and Fig. 7. Same phenomena with Fig. 7 are observed in the left side area and the right side area. Same phenomena with Fig. 5 are observed in the center area. Figure 11 shows a computer simulation result in the case of boundary area between Fig. 6 and Fig. 7. This case looks like a high frequency state of Fig. 6. However, switching timing is not in time. It is also shown in green line of Fig 11 (b). Figure 12 shows a computer simulation result in the case of an anti-phase synchronization of switching phenomena with synchronization phenomena. In this case, synchronization phenomena are observed between anti-phase synchronizations of switching phenomena. This synchronization time is corresponding to parameter δ mainly.

B. Relationship between parameters and phenomena

Next, we investigate the relationship between parameters and phenomena which is mentioned above. Figure 13 shows a relationship among parameter β_2, δ and phenomena. Parameters β_2 and δ are corresponding to the inductor L_2 and the resistor G which is coupling strength. The relationship between colors and observed phenomena is shown in Table I. Following points could be mentioned from Fig. 13.

- In-phase and anti-phase synchronizations of switching phenomena are observed between anti-phase synchronization states as shown in Fig. 7 and the state of holding one kinds of attractors as shown in Fig. 9.
- Anti-phase synchronizations of switching phenomena are observed in the case of $\beta_1 = \beta_2$
- In-phase synchronizations of switching phenomena are observed in the case that β_2 is differ from β_1.
- In the case that β_2 is nearly equal to 12, in-phase and anti-phase synchronizations of switching phenomena are observed by changing δ only.

Additionally, we confirmed that by increasing a value of δ, sojourn time of until switching becomes long in the case of in-phase and anti-phase synchronization of switching phenomena. We consider that in the case of increasing the number of the circuit, anti-phase synchronizations of switching phenomena are observed and in-phase synchronizations of switching phenomena are not observed. Because anti-phase can be observed in the case of $\beta_1 = \beta_2$ and in-phase cannot be observed in the case of $\beta_1 = \beta_2$ or $\beta_1 \simeq \beta_2$. Namely, the difference of β_n is not ignore in the case of increasing the number of the circuit.

Figure 14 shows a relationship among parameters α_2, δ and phenomena. Parameters α_2 is corresponding to negative resistor r_2. The relationship between colors and observed phenomena is shown in Table I. Anti-phase synchronizations

TABLE I

COLOR DEFINITION OF FIG. 13 AND FIG. 14.

Fig.	State
Fig. 5	Red
Fig. 6	Green
Fig. 7	Blue
Fig. 8	Light blue
Fig. 9	Gray
Fig. 10	Pink
Fig. 11	Light green
Fig. 12	Yellow

of switching phenomena are only observed. The area is around $0.55 \le \delta \le 0.63$ and $0.46 \le \alpha_2 \le 0.48$. Namely, anti-phase synchronization is not observed in the case of $\alpha_1 \simeq \alpha_2$ or a small value of coupling factor δ. In yellow areas as shown in Fig. 14, by decreasing α_2, synchronization time becomes long.

IV. CONCLUSIONS

In this study, we have investigated coupled chaotic circuits and have observed in-phase and anti-phase synchronizations of switching phenomena. In the case that the number of circuit is two, the relationship between parameters and phenomena is shown.

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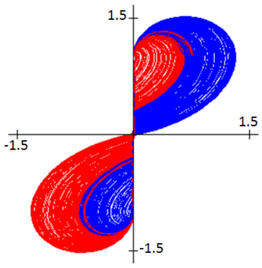

Fig. 4. Coexisting attractors of a chaotic circuit as shown in Fig. 1. $\alpha_1 = 0.40$, $\beta_1 = 3.0$ and $\gamma = 470.0$.

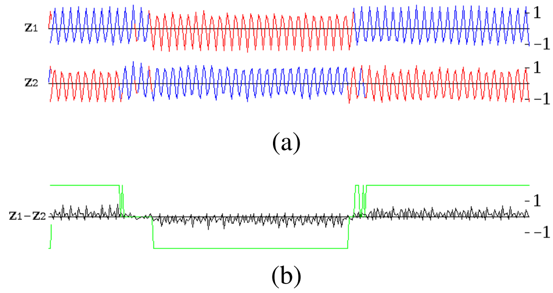

Fig. 5. Anti-phase synchronization of switching phenomena. $\alpha_n = 0.40$, $\beta_1 = 3.0$, $\beta_2 = 4.5$, $\gamma = 470.0$ and $\delta = 0.31$.

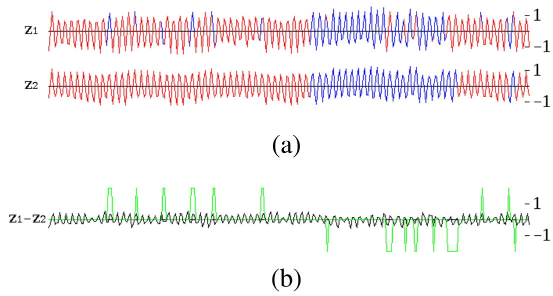

Fig. 6. In-phase synchronization of switching phenomena. $\alpha_n = 0.40$, $\beta_1 = 3.0$, $\beta_2 = 15.0$, $\gamma = 470.0$ and $\delta = 0.33$.

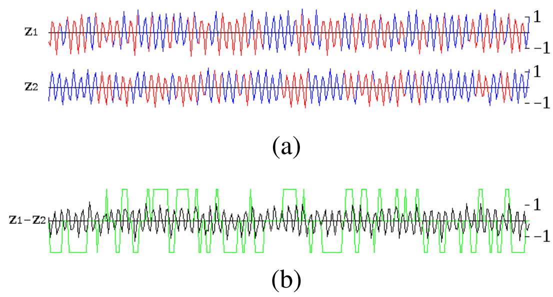

Fig. 7. Asynchronization of switching phenomena. $\alpha_n = 0.40$, $\beta_1 = 3.0$, $\beta_2 = 9.0$, $\gamma = 470.0$ and $\delta = 0.10$.

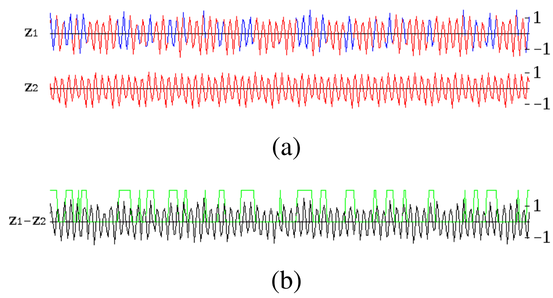

Fig. 8. Switching phenomena on one circuit only. $\alpha_n = 0.40$, $\beta_1 = 3.0$, $\beta_2 = 12.0$, $\gamma = 470.0$ and $\delta = 0.10$.

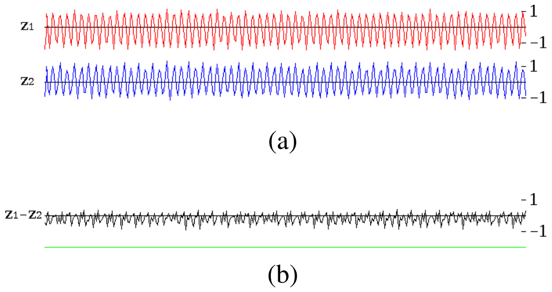

Fig. 9. Holding one attractor state in each circuit. $\alpha_n = 0.40$, $\beta_1 = 3.0$, $\beta_2 = 7.5$, $\gamma = 470.0$ and $\delta = 0.25$.

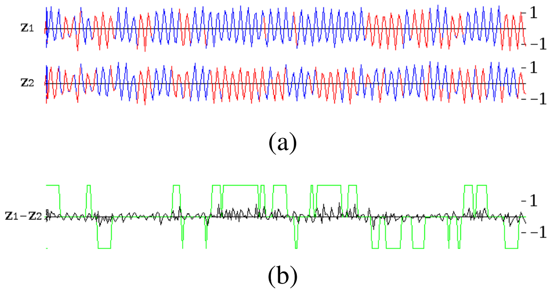

Fig. 10. Boundary area between Fig. 5 and Fig. 7. $\alpha_n = 0.40$, $\beta_1 = 3.0$, $\beta_2 = 6.0$, $\gamma = 470.0$ and $\delta = 0.22$.

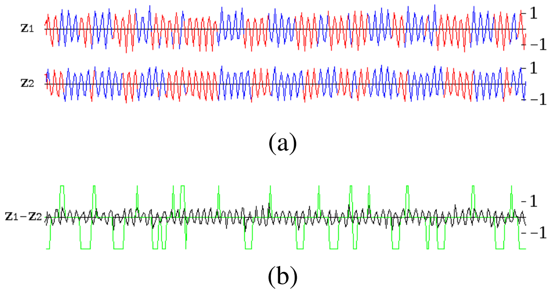

Fig. 11. Boundary area between Fig. 6 and Fig. 7. $\alpha_n = 0.40$, $\beta_1 = 3.0$, $\beta_2 = 18.0$, $\gamma = 470.0$ and $\delta = 0.21$.

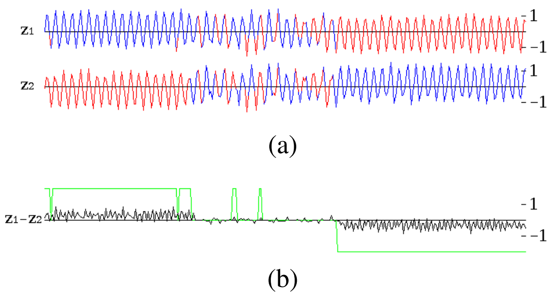

Fig. 12. Anti-phase synchronization of switching phenomena with synchronization phenomena. $\alpha_1 = 0.40$, $\alpha_2 = 0.46$, $\beta_n = 3.0$, $\gamma = 470.0$ and $\delta = 0.52$.

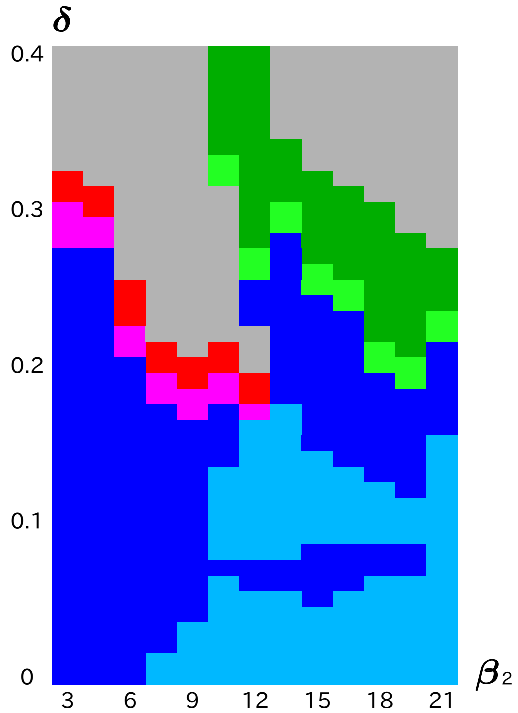

Fig. 13. Relationship among parameter β_2, δ and observed phenomena. $\alpha_n = 0.40$, $\beta_1 = 3.0$ and $\gamma = 470.0$.

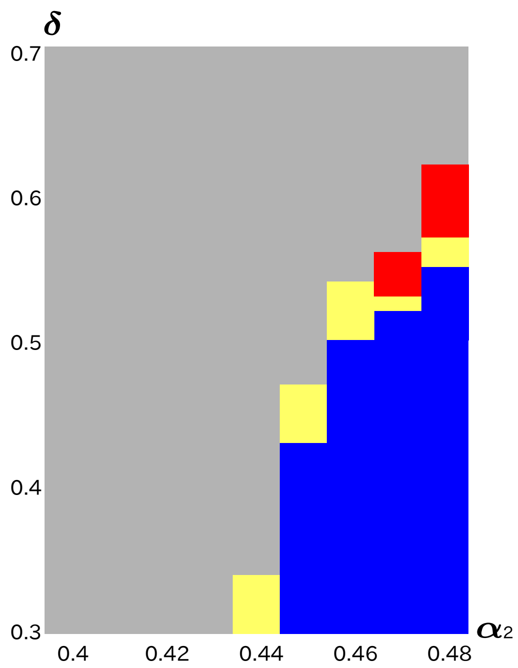

Fig. 14. Relationship among parameter α_2, δ and observed phenomena. $\alpha_1 = 0.40$, $\beta_n = 3.0$ and $\gamma = 470.0$.