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Synchronisation in Four Coupled van der Pol Oscillators in a Regular Tetrahedron Form

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1. Introduction

High dimensional nonlinear phenomena can often be expressed by synchronization phenomena of oscillators. Therefore, studies of the synchronization phenomena of coupled oscillators are reported in many research fields. However, synchronization phenomena of the oscillators have not been analyzed enough yet.

In this study, we propose four coupled van der Pol oscillators in a regular tetrahedron form. We investigate synchronisation phenomena in this circuit model.

2. Circuit Model

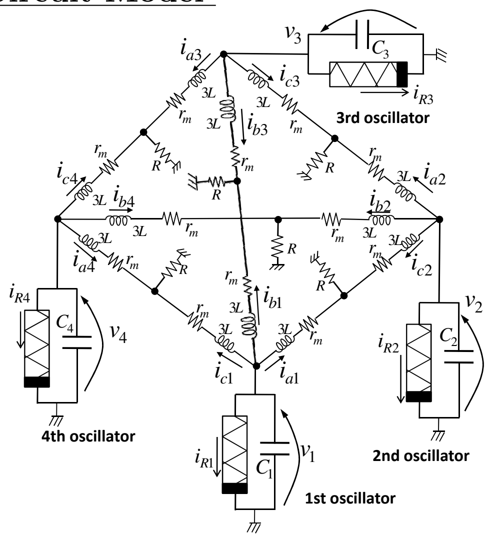

Figure 1: Four coupled van der Pol oscillators.

The circuit model of four coupled van der Pol oscillators in a regular tetrahedron form is shown in Fig. 1. Van der Pol oscillator consists of an inductor, a negative resistor and a capacitor. In addition, we assume that the $v_k - i_{Rk}$ characteristics of nonlinear resistor in each oscillator is given by the following third order polynomial equation.

$$i_{Rk} = -g_1 v_k + g_3 {v_k}^3 \qquad (g_1, g_3 > 0), \quad (k = 1, 2, 3, 4). \tag{1}$$

The normalized circuit equations are expressed as:

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon(1 - {x_k}^2)x_k - (y_{ak} + y_{bk} + y_{ck}) \\ \dfrac{dy_{ak}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{ak} - \gamma(y_{ak} + y_{c(k+1)})\right\} \\ \dfrac{dy_{bk}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{bk} - \gamma(y_{bk} + y_{b(k+2)})\right\} \\ \dfrac{dy_{ck}}{d\tau} = \dfrac{1}{3}\left\{x_k - \eta y_{ck} - \gamma(y_{ck} + y_{a(k+3)})\right\}. \end{cases} \tag{2}$$

We used the following normalizations:

$$t = \sqrt{LC}\tau \ , v_k = \sqrt{g_1/g_3}\, x_k \ , i_{ak} = \sqrt{g_1 C/g_3 L}\, y_{ak},$$

$$i_{bk} = \sqrt{g_1 C/g_3 L}\, y_{bk} \ , i_{ck} = \sqrt{g_1 C/g_3 L}\, y_{ck},$$

$$\varepsilon = g_1\sqrt{L/C} \ , \gamma = R\sqrt{C/L} \ , \eta = r_m\sqrt{C/L},$$

where ε is the degree of nonlinearity, γ is the coupling strength, and η indicates the resistive component.

3. Synchronization phenomena

We carry out computer simulations for the four coupled van der Pol oscillators in a regular tetrahedron form circuit. We observe a synchronization phenomena that phase differences between oscillators shift from 0° to 180° or the reverse, periodically. The synchronization states stay at the anti-phase whose sojourn time is longer than that of the in-phase. Fig. 2 shows examples of phase differences for the parameters $\varepsilon = 1.0$, $\eta = 0.02$ and $\gamma = 0.15$ (Fig. 2(a)) or $\gamma = 0.40$ (Fig. 2(b)). In Fig. 2, "1-2" indicates a phase difference between 1st oscillator and 2nd oscillator. Periods of the shifting the phase differences become longer with increasing γ.

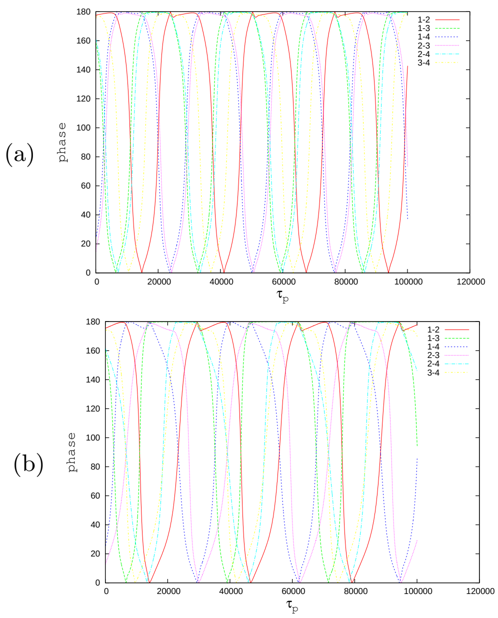

Figure 2: Phase difference between oscillators for $\varepsilon = 1.0$ and $\eta = 0.02$. (a)$\gamma = 0.15$. (b)$\gamma = 0.40$.

4. Conclusions

In this study, we proposed the four coupled oscillators in a regular tetrahedron form. We observed synchronisation phenomena that the phase difference changed periodically. Additionally, when the coupled strength was small, changes of phase differences became sharp.

References

[1] Y. Uwate, Y. Nishio and R Stoop, "Synchronization in Three Coupled van der Pol Oscillators with Different Coupling Strength",Proceedings of RISP International Workshop on Nonlinear Circuits and Signal Processing (NCSP'10), pp. 109-112, Mar. 2010.