

Synchronization of Two Chaotic Circuits Coupled via Mutual Inductance

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1. Introduction

Synchronization phenomenon is one of the fundamental phenomena in nature. Especially, chaos synchronization phenomena in nonlinear circuits have been studied in many fields. Thus, it is thought that investigations of the synchronization phenomena will be important in the future.

In this study, we investigate synchronization phenomena in two coupled chaotic circuits via mutual inductance. We observe some interesting synchronization phenomena.

2. Circuit model

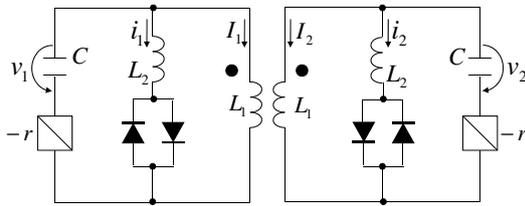


Figure 1: Circuit model.

The circuit model used in this study is shown in Fig. 1. In this circuit, two identical chaotic circuits are coupled via mutual inductance.

First, we approximate the $i - v$ characteristics of the nonlinear resistor consisting of diodes by the following function.

$$v_d(i_k) = \sqrt[3]{r_d i_k}. \quad (1)$$

By changing the variables and parameters,

$$\begin{aligned} I_k &= a\sqrt{C/L_1}x_k, \quad i_k = \sqrt{C/L_1}y_k, \quad v_k = az_k, \\ \alpha &= \sqrt{L_1/L_2}, \quad \beta = \sqrt{C/L_1}, \quad \gamma = M/L_1, \\ t &= \sqrt{L_1 C} \tau, \quad \text{"."} = d/d\tau, \\ &\left(\text{where } a = \sqrt[8]{r_d \sqrt{C/L_1}} \right) \end{aligned} \quad (2)$$

the circuit equations are normalized as

$$\begin{cases} \dot{x}_1 = \frac{1}{1-\gamma^2} \{ \beta(x_1 + y_1) - z_1 \} \\ \quad - \frac{\gamma}{1-\gamma^2} \{ \beta(x_2 + y_2) - z_2 \} \\ \dot{y}_1 = \alpha \{ \beta(x_1 + y_1) - z_1 - f(y_1) \} \\ \dot{z}_1 = x_1 + y_1 \end{cases} \quad (3)$$

$$\begin{cases} \dot{x}_2 = -\frac{\gamma}{1-\gamma^2} \{ \beta(x_1 + y_1) - z_1 \} \\ \quad + \frac{1}{1-\gamma^2} \{ \beta(x_2 + y_2) - z_2 \} \\ \dot{y}_2 = \alpha \{ \beta(x_2 + y_2) - z_2 - f(y_2) \} \\ \dot{z}_2 = x_2 + y_2 \end{cases} \quad (4)$$

3. Simulation Results

We carry out computer calculations for the two chaotic circuits coupled via mutual inductance. We observe various synchronization phenomena. Figure 2 shows the observed attractors, phase differences and time waveform for the parameters (a): $\alpha = 24.0$, $\beta = 0.20$ and $\gamma = 0.60$, (b): $\alpha = 24.0$, $\beta = 0.30$ and $\gamma = 0.05$. In Fig. 2(a), the two chaotic circuits are synchronized at the opposite-phase. Whereas, the two chaotic circuits are synchronized at the in-phase as shown in Fig. 2(b). In the coupled circuits, the two circuits are synchronized at the opposite-phase when β , which determines chaotic characteristics of each circuit, is small. While, the two circuits are synchronized at the in-phase when β is large.

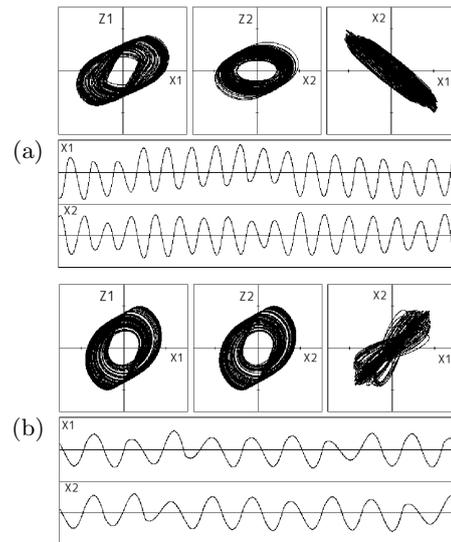


Figure 2: Opposite-phase synchronization and in-phase synchronization are observed for the values of β which are small values and large values, respectively. $\alpha = 24.0$. (a) $\beta = 0.20$ and $\gamma = 0.60$. (b) $\beta = 0.30$ and $\gamma = 0.05$.

4. Conclusions

In this study, we investigated the synchronization phenomena in chaotic circuits coupled via mutual inductance. By carrying out computer calculations for the circuits, we confirmed various kinds of synchronization phenomena. In-phase synchronization and opposite-phase synchronization can be observed for different parameters.

References

[1] Y. Nishio and A. Ushida, "Spatio-Temporal Chaos in Simple Coupled Chaotic Circuits," IEEE Transactions on Circuits and Systems I, vol. 42, no. 10, pp. 678-686, Oct. 1995.