

## ACO with Intelligent and Dull Ants for QAP

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### 1. Introduction

Ant Colony Optimization (ACO) [1] is a biologically inspired optimization algorithm with pheromone effects of ants and is effective to solve difficult combinatorial optimization problems like the Quadratic Assignment Problem (QAP). QAP is one of the NP-hard combinatorial optimization problems.

In our previous study, we have proposed a new ACO algorithm; ACO with Intelligent and Dull ants (IDACO) [2]. In IDACO, two kinds of ants coexist; *intelligent ant* and *dull ant*. The intelligent ant can trail the pheromone, however the dull ants cannot trail the pheromone. We have confirmed that IDACO obtained effective result for the Traveling Salesman Problem. In this study, we apply IDACO to various QAPs.

### 2. IDACO

Given two matrices, a distance matrix  $\mathbf{D}$  and a flow matrix  $\mathbf{F}$ , one sums the elements of each line to obtain two vectors  $\mathcal{D}$  and  $\mathcal{F}$ , find a permutation  $\Pi$  which corresponds to the minimum value of the total assignment cost  $L$  in Eq. (1).

$$L = \sum_{i=1}^n \sum_{j=1}^n D_{ij} F_{\pi(i)\pi(j)}, \quad (1)$$

where  $D_{ij}$  and  $F_{ij}$  are the  $(i, j)$ -th elements of  $\mathbf{D}$  and  $\mathbf{F}$ , respectively,  $\pi(i)$  is the  $i$ -th element of the vector  $\Pi$ , and  $n$  is the size of the problem. The number of ants is denoted by  $M$ .  $d \times M$  ants are classified into a set of the dull ants  $S_{\text{dull}}$ .  $d$  is a rate of dull ants in all the ants.

[Step1] Let  $t = 0$ .  $\tau_{ij}(t)$  is the amount of pheromone trail on a coupling  $(i, j)$  to assign the activity  $j$  to the location  $i$ , and  $\tau_{ij}(0) = \tau_0$ .

[Step2] From the two vectors  $\mathcal{D}$  and  $\mathcal{F}$ , one may obtain a matrix  $\mathbf{A}$ . The element  $a_{ij}$  of  $\mathbf{A}$  is given by the product  $d_i \cdot f_j$ . The probability that  $k$ -th ant ( $k = 1, \dots, M$ ) assigns activity  $j$  to location  $i$  is decided by

$$p_{k,ij}(t) = \begin{cases} \frac{[\eta_{ij}]^{\beta_D}}{\sum_{l \in N^k} [\eta_{il}]^{\beta_D}}, & \text{if } k \in S_{\text{dull}} \\ \frac{[\tau_{ij}(t)]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in N^k} [\tau_{il}(t)]^\alpha [\eta_{il}]^\beta}, & \text{otherwise,} \end{cases} \quad (2)$$

where  $1/\eta_{ij}$  is the element  $a_{ij}$ . The adjustable parameters  $\alpha$  and  $\beta$ ,  $\beta_D$  control the weight of pheromone intensity and of the element, respectively.  $N^k$  is a set of activities such that  $k$ -th ant has not yet assigned any activity in the set.

[Step3] After all the ants have completed assignment, compute the total cost  $L_k(t)$  and update the amount of the pheromone  $\tau_{ij}(t)$ . Note that dull ants can deposit the pheromone, though, they cannot trail the pheromone. The amount of the pheromone  $\Delta\tau_{ij}^k$  deposited by  $k$ -th ant on the coupling  $(i, j)$  is decided as

$$\Delta\tau_{ij}^k(t) = \begin{cases} 10/L^k, & \text{if } (i, j) \in T^k(t) \\ 0, & \text{otherwise,} \end{cases} \quad (3)$$

where  $T_k(t)$  is obtained permutation by  $k$ -th ant and  $L_k(t)$  is its total cost.  $\rho \in [0, 1]$  is the pheromone trail decay coefficient. Update  $\tau_{ij}(t)$  of each coupling  $(i, j)$  depending on its  $\Delta\tau_{ij}^k$ ;

$$\tau_{ij}(t+1) = \rho\tau_{ij}(t) + \sum_{k=1}^M \Delta\tau_{ij}^k(t). \quad (4)$$

[Step4] Let  $t = t + 1$ . Go back to [Step2] and repeat until the maximum time limit  $t = t_{\text{max}}$ .

### 3. Numerical Experiments

We apply IDACO to three QAPs and compare IDACO with different rates of dull ants with the standard ACO.

In the experiments, the number of ants  $M$  is set to the same as the number of locations. The rate of dull ants  $d$  is set to 0.2 and 0.5 in each simulation. We repeat the simulation 20 times for all the problems. The parameters are set as follows;

$$\tau_0 = 10, \rho = 0.9, \alpha = 1, \beta = \beta_D = 1, t_{\text{max}} = 15000.$$

The simulation results are shown in Table 1. We can confirm that IDACO obtains better results than the standard ACO. Especially, IDACO with  $d = 0.5$  obtains the best results. We can confirm that IDACO including more dull ants obtains the effective results.

Table 1: Results of the standard ACO and IDACO.

|                  | Scr12    | Tai12     | Nug12 |
|------------------|----------|-----------|-------|
| The standard ACO | 34067.7  | 2472283.1 | 613.2 |
| IDACO with 0.2   | 338488.5 | 245435.8  | 615   |
| IDACO with 0.5   | 33846.5  | 247031    | 611.9 |
| Optimal solution | 31410    | 224416    | 578   |

### 4. Conclusions

This study has applied IDACO to quadratic assignment problems. We have confirmed that IDACO obtained better results than the standard ACO because the dull ants help in getting out of the local optima.

### References

- [1] M. Dorigo and T. Stutzle, *Ant Colony Optimization*, Bradford Books, 2004.
- [2] S. Shimomura, M. Sugimoto, T. Haraguchi, H. Matsushita, and Y. Nishio, "Behavior of Ant Colony Optimization with Intelligent and Dull Ants", *Technical Report on Nonlinear Problems*, no. NLP2010-20 & NC2010-20, pp. 157-160, Jun. 2010.