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WC-SOM and its Flexible Self-Organization

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***Abstract*—This study proposes weighted connections avoiding false-neighbor effects for the Self-Organizing Map. The new SOM is called WC-SOM. We investigate the effectiveness of WC-SOM in comparison with the conventional SOM, Growing Grid and FN-SOM. We confirm that WC-SOM enables the most flexible self-organization among four algorithms and can obtain the effective map reflecting the distribution state of the input data using fewer neurons.**

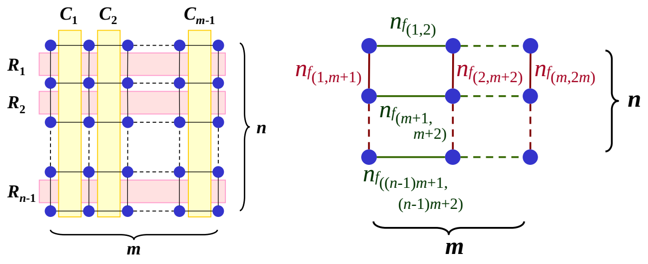
I. Introduction

The Self-Organizing Map (SOM) is an unsupervised neural network [1] and has attracted attention for its clustering properties. In the learning algorithm of SOM, a winner, which is a neuron closest to an input data, and its neighboring neurons on the map are updated, regardless of the distance between the input data and the neighboring neurons. For this reason, if we apply SOM to clustering of the input data including some clusters located at distant location, there are some inactive neurons between clusters where without the input data. Because the inactive neurons are on a part, we are misled into thinking that there are some input data between clusters. Furthermore, because the simulation time depends on the number of neurons, it is important to utilize the used neurons effectively by reducing the inactive neurons.

In the real world, it is not always true that neighboring houses are physically adjacent or close to each other. In other words, "neighbors" are not always "true neighbors". In addition, the relationship between neighborhoods is not fixed, but keeps changing with time. It is important to change the neighborhood relationship flexibly according to the situation.

Meanwhile, the synaptic strength is not constant in the brain. So far, the Growing Grid network was proposed in 1985 [2]. Growing Grid increases the neighborhood distance between neurons by increasing the number of neurons. However, there is not much research changing the synaptic strength.

In our past study, we proposed the SOM with False-Neighbor degree between neurons (called FN-SOM) [4]. False-neighbor degrees (FNDs) are allocated between adjacent rows and adjacent columns of FN-SOM. FNDs act as a burden of the distance between map nodes when the weight vectors of neurons are updated. FN-SOM can greatly reduce the inactive neurons, however, the algorithm has following problems. All the FNDs between the neurons on same line are increased simultaneously and forcibly. It often produces the increase of FNDs between the correct-neighboring neurons, namely, it is defined as a FALSE false-neighbor.



(a) (b)

Fig. 1. Differences between false-neighbor degree of FN-SOM and false-neighbor weights of WC-SOM. Neurons of both SOMs are located at a $n \times m$ rectangular grid. (a) A false-neighbor degree of row R_r $(1 \leq r \leq n-1)$ and column C_k $(1 \leq k \leq m-1)$. (b) False-neighbor weights $n_{f(i,k)}$ are allocated between all the directly-connected neurons i and k. $k \in N_{i1}$

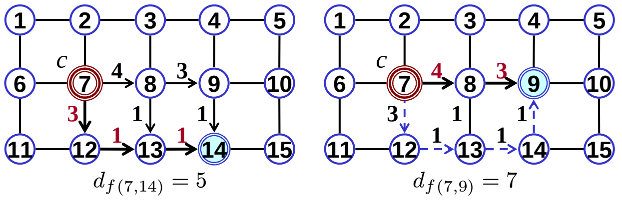
In this study, we incorporate the weighted connections avoiding false-neighbor effects into SOM. The new SOM is called SOM with Weighted Connections avoiding false-neighbor effects (WC-SOM). In WC-SOM, all the connections between adjacent neurons are weighted to avoid false-neighbor effects unlike FN-SOM. This weights are called as false-neighbor weights (FNWs). In the algorithm of WC-SOM, we find the winless neurons and its "false neighbors", and FNWs between these neurons are increased. In other words, by increasing FNWs with learning, WC-SOM changes the neighborhood relationship more flexibly according to the situation and the shape of data.

We compare WC-SOM with the conventional SOM, Growing Grid and FN-SOM, and effectiveness of WC-SOM are investigated by applying to complex input data. We confirm that WC-SOM can obtain the effective map reflecting the distribution state of the input data using fewer neurons.

II. SOM with Weighted Connections avoiding false-neighbor effects (WC-SOM)

The conventional method, FN-SOM, has false-neighbor degrees (FNDs) between neurons shown as Fig. 1(a). However, all the FNDs between neurons at same line on the map are forcibly the same value. It often produces the twist of the map.

In this study, we propose Weighted Connections avoiding false-neighbor effects for SOM (WC-SOM). We explain WC-SOM in detail in this section. WC-SOM consist of $n \times m$ neurons located at 2-dimensional rectangular grid. Each neuron i has a d-dimensional weight vector $\boldsymbol{w}_i = (w_{i1}, w_{i2}, \cdots, w_{id})$



(a) (b)

Fig. 2. The connection weight $d_{f(c,i)}$ between the winner c and the neuron i. $d_{f(c,i)}$ is a minimum sum-of-FNWs of neurons connected by the shortest-path distance from c to i. Values in the circle mean respective neuron numbers. Values between connections mean FNWs n_f between directly-connected neurons. The winner c is 7. (a) Let $i = 14$. The possible shortest-path routes from c to i are [7 8 9 14], [7 8 13 14] and [7 12 13 14]. The sum of FNWs of respective routes are $n_{f(7,8)} + n_{f(8,9)} + n_{f(9,14)} = 8$, $n_{f(7,8)}+n_{f(8,13)}+n_{f(13,14)} = 6$ and $n_{f(7,12)}+n_{f(12,13)}+n_{f(13,14)} = 5$. Therefore, $d_{f(7,14)} = 5$ which is minimum value. (b) Let $i = 9$. $d_{f(7,9)} = 7$ because the possible shortest-path route is [7 8 9] only. Note that a route [7 12 13 14 9] is NOT adopted because it is not the shortest-path route although the sum of FNWs of this route is 6 which is smaller than 7.

$(i = 1, 2, \cdots, nm)$. The range of the elements of the input data $\boldsymbol{x}_j = (x_{j1}, x_{j2}, \cdots, x_{jd})$ $(j = 1, 2, \cdots, N)$ are assumed to be from 0 to 1.

In WC-SOM, a false-neighbor weight $n_{f(i,k)}$ (FWN) is allocated between directly-connected neurons i and k shown as Fig. 1(b), and we propose the new neighborhood distance considering FNWs. The initial value of each FNW n_f between directly-connected neurons is set to zero, and the initial values of all the weight vectors are given over the input space at random. Moreover, a winning frequency γ_i is associated with each neuron and is set to zero initially.

Learning Step

(Step1) Input an input vector $\boldsymbol{x}_j$ to all the neurons.

(Step2) Calculate distances between $\boldsymbol{x}_j$ and all the weight vectors. Find a winner, denoted by c, which is the neuron with the weight vector closest to the input vector $\boldsymbol{x}_j$:

$$c = \arg\min_i \{\|\boldsymbol{w}_i - \boldsymbol{x}_j\|\}, \tag{1}$$

where $\|\cdot\|$ is Euclidean distance measure.

(Step3) Increase the winning frequency of the winner c by $\gamma_c^{\text{new}} = \gamma_c^{\text{old}} + 1$.

(Step4) Calculate the neighboring distance $dis(c, i)$ between the winner c and each neuron i by considering FNWs n_f as the following measure ;

$$dis(c, i) = \|\boldsymbol{r}_i - \boldsymbol{r}_c\|^2 + d_{f(c,i)}, \tag{2}$$

where $\|\boldsymbol{r}_i - \boldsymbol{r}_c\|$ is the Euclidean distance between map nodes c and i on the map grid. $d_{f(c,i)}$ is the connection weight between c and i, and it is defined as a minimum sum-of-FNWs of the neurons connected by the shortest-path distance from c to i (as Fig. 2).

(Step5) Update the weight vectors of all the neurons:

$$\boldsymbol{w}_i(t + 1) = \boldsymbol{w}_i(t) + h_{Fc,i}(t)(\boldsymbol{x}_j - \boldsymbol{w}_i(t)), \tag{3}$$

where $h_{Fc,i}(t)$ is the neighborhood function of WC-SOM:

$$h_{Fc,i}(t) = \alpha(t) \exp\left(-\frac{dis(c, i)}{2\sigma^2(t)}\right). \tag{4}$$

where $\alpha(t)$ is the learning rate, and $\sigma(t)$ corresponds to the width of the neighborhood function. Both $\alpha(t)$ and $\sigma(t)$ decrease with time, in this study, we use following equations;

$$\alpha(t) = \alpha_0(1 - t/t_{\max}), \quad \sigma(t) = \sigma_0(1 - t/t_{\max}), \tag{5}$$

where α_0 and σ_0 are the initial values of α and σ, respectively, and t_{max} is the maximum number of the learning.

(Step6) If $\sum_{i=1}^{nm} \gamma_i \geq \lambda$ is satisfied, find the false-neighbors and increase the false-neighbor weights n_f, according to steps from (Step7) to (Step10). If not, perform step (Step11). In other words, we consider the false-neighbors every time when the learning steps are performed for λ input data.

Considering False-Neighbors Step

(Step7) Find a set of neurons S which have never become the winner: $S = \{i \mid \gamma_i = 0\}$. If the winless neuron does not exist, namely $S = \emptyset$, return to (Step1) without considering the false-neighbors.

(Step8) Choose a false-neighbor f_q of each neuron q in S from the set of direct topological neighbors of q denoted as N_{q1}. f_q is the neuron whose weight vector is most distant from q:

$$f_q = \arg\max_i \{\|\boldsymbol{w}_i - \boldsymbol{w}_q\|\}, \quad q \in S, \ i \in N_{q1}. \tag{6}$$

(Step9) Increase FNW $n_{f(q,f_q)}$ between each q and its false-neighbor f_q as

$$n_{f(q,f_q)} = n_{f(q,f_q)} + 1, \tag{7}$$

where $n_{f(q,f_q)} = n_{f(f_q,q)}$.

(Step10) Reset the winning frequency of all the neurons to zero: $\gamma_i = 0$.

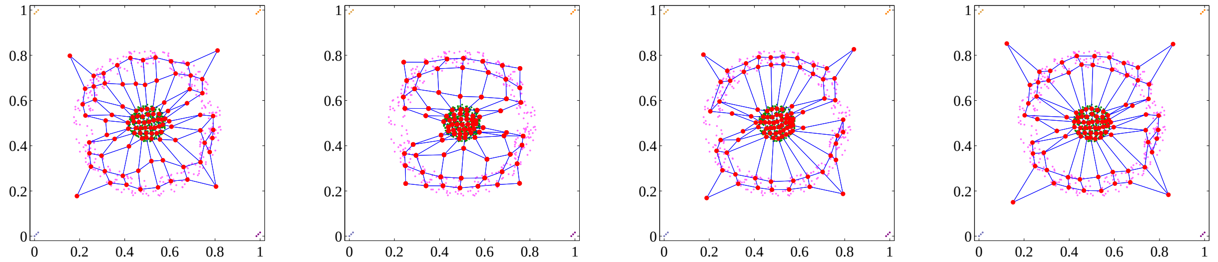
(Step11) Repeat the steps from (Step1) to (Step10) for all the input data.

III. Experimental Results

We apply WC-SOM to complex data and compare it with the conventional SOM, Growing Grid and FN-SOM. For the experiment, SOM, FN-SOM and WC-SOM have $nm = 100$ neurons (10×10). Growing Grid starts learning with 2×2 neurons, and the maximum number of neurons is less than 100. The parameters of the learning for WC-SOM are chosen as $\alpha_0 = 0.3$, $\sigma_0 = 3$, $\lambda = 3000$, where we use the same α_0 for all the method and the same λ for FN-SOM and WC-SOM. For the conventional SOM, Growing Grid and FN-SOM, we use $\sigma_0 = 4$.

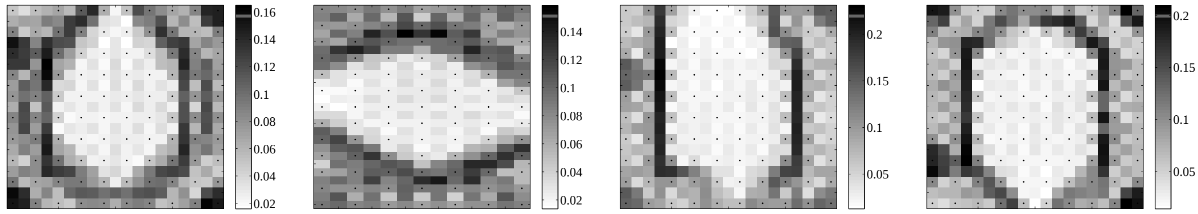
A. For Target data set

We consider Target data set [5] which has a clustering problem of outliers. The input data is 2-dimension and has six clusters including 4 outliers, and the total number of the



(a) (b) (c) (d)

Fig. 3. Learning results of four algorithms for Target data. (a) Conventional SOM. (b) Growing Grid. (c) FN-SOM. (d) WC-SOM.



(a) (b) (c) (d)

Fig. 4. U-Matrices of simulation results for Target data. (a) Conventional SOM. (b) Growing Grid. (c) FN-SOM. (d) WC-SOM.

input data N is 770. We repeat the learning 26 times for all the input data, namely $t_{\max} = 20020$.

Learning results of the conventional SOM, Growing Grid are shown in Figs. 3(a) and (b). We can see that there are a lot of inactive neurons between clusters. The other side, from the learned maps of FN-SOM and WC-SOM shown in Figs. 3(c) and (d), it is clear that FN-SOM and WC-SOM can greatly reduce the number of inactive neurons in comparison with the conventional SOM.

Furthermore, in order to compare the learning performance of WC-SOM with the other SOMs numerically, we use the following well-used two measurements to evaluate the training performance.

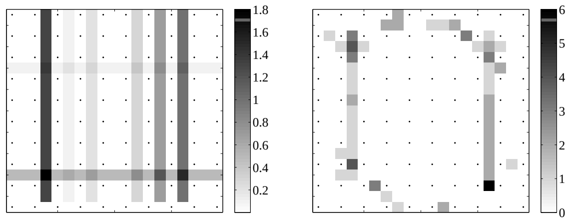
Quantization Error Qe measures the average distance between each input vector and its winner [1]. The small value Qe is more desirable.

Neuron Utilization U measures the percentage of neurons that are the winner of one or more input vector in the map [3]. Thus, U nearer 1.0 is more desirable.

TABLE I

TWO MEASUREMENTS FOR TARGET DATA.

	SOM	Growing Grid	FN-SOM	WC-SOM
Qe	0.0196	0.0232	0.0190	0.0175
U	0.8113	0.8317	0.9263	0.8917



(a) (b)

Fig. 5. Gray scale display of false-neighbor degrees and false-neighbor weights. (c) False-neighbor degrees of FN-SOM. (d) False-neighbor weights of WC-SOM.

We carry out the 30 simulations with different initial state of the weight vectors and different order of inputting. The averages of two measurements over 30 independent runs are listed in Table I. It should be noted that the quantization error Qe of WC-SOM is better than FN-SOM although the neuron utilization U of FN-SOM is the best value. This means that there are a few inactive neurons in the result of FN-SOM, however, FN-SOM has not self-organized the statistical features of input data correctly. In other words, WC-SOM can obtain the clustering map reflecting the distribution state of the input data more effectively than FN-SOM.

In order to evaluate how well SOM preserves the topology of the data set, we calculate U-Matrix [5] which visualizes the cluster structure of the map by showing distances between the neighboring neurons. Figure 4 shows U-Matrices of four algorithms. We can see that the boundary lines of FN-SOM and WC-SOM are clearer than other two algorithms, and it is easy to distinguish between light areas (cluster) and dark area (no input data) because there are few inactive neurons in these results. However, it should be noted that it is clear that U-Matrix of FN-SOM does not visualize the cluster structure correctly. Meanwhile, U-Matrix of WC-SOM visualises the cluster structure effectively, although its boundary lines is very clear. It means that FN-SOM can not preserve the topology of the data set by increasing its FNDs forcibly, and WC-SOM can obtain the clustering map reflecting the distribution state of the input data more effectively than FN-SOM.

This can be confirmed by the difference between FNDs of FN-SOM and FNWs of WC-SOM after learning. Figure 5 shows FNDs and FNWs between the neurons displayed by gray-scale. From FNDs of FN-SOM as Fig. 5(a), we can clearly see that even FNDs between the CORRECT-neighboring neurons have been increased. This is because that all the FNDs between the neurons on same line are increased simultaneously and forcibly, in FN-SOM algorithm. On the other hand, in Fig. 5(b), FNWs of WC-SOM has be increased flexibly depending on the shape of the input data in comparison with FN-SOM.

IV. Conclusions

This study incorporates the weighted connections avoiding false-neighbor effects into SOM (WC-SOM). WC-SOM has false-neighbor weights (FNWs) allocated between connections of adjacent neurons to avoiding false-neighbor effect. In the algorithm, the neighborhood distance has been proposed in accordance with FNWs. We have compare WC-SOM with the conventional SOM, Growing Grid and FN-SOM and have confirmed that WC-SOM enables the most flexible self-organization by increasing FNWs with learning. Furthermore, we have confirmed visually and numerically that WC-SOM has few inactive neurons. In addition, FNWs of WC-SOM has be increased flexibly depending on the shape of the input data in comparison with FN-SOM. These results mean that WC-SOM can obtain the effective map reflecting the distribution state of the input data using less neuron.

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