Peak Search and Tracing Algorithm for Frequency Analysis of Nonlinear Circuits

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***Abstract*—In this article, we propose a SPICE-oriented algorithm to analyze frequency characteristics with unstable region and to find the peaks of the frequency characteristics. By combining the sine-cosine circuits, the Fourier transformation circuit, the nonlinear limiter and the solution tracing circuit, the frequency characteristics curve can be obtained even if the curve has unstable region.**

I. Introduction

For designing PCBs (printed circuit boards), it is very important to find out the locations and the frequencies giving large peak values of the voltages. Electrostatic capacity exists between the wire lines of PCBs. The characteristics of the capacitors depend on the distance between the wire lines. From this reason, we have to analyze circuits including nonlinear capacitors. The SPICE can treat nonlinear elements, however the standard method of SPICE cannot obtain frequency characteristics of nonlinear circuits easily.

In this study, we propose a SPICE-oriented algorithm to analyze frequency characteristics of nonlinear circuits and to find the peaks of the frequency characteristics. Although they may be found by the standard transient analysis of SPICE, it is difficult to find the exact peaks when the quality factor (Q) is very large. Because we may pass over them if we choose a large step size. Furthermore, the frequency characteristics of nonlinear circuits often have unstable regions. Such regions cannot be obtained the standard methods using SPICE. In our algorithm, we derive the sine-cosine circuit [1] from the nonlinear circuit. Next, the Fourier transformation circuit [2] is used to obtain the response of nonlinear elements. When we analyze this circuit with the transient analysis of SPICE, we may pass over the exact peaks. In order to avoid this problem, we apply the differentiator and the nonlinear limiter [3]. Finally, we apply the STC (solution trace circuit) [4] to obtain the frequency characteristics even when the curve has unstable regions.

II. SPICE-Oriented Harmonic Balance Algorithm

A. Sine-cosine transformation

Sine-cosine transformation based on the HB (harmonic balance) method such that the determining equation is solved by transient analysis of SPICE. We discuss the sine-cosine circuit corresponding to the determining equation of the HB method. If we set the voltage through a capacitor C

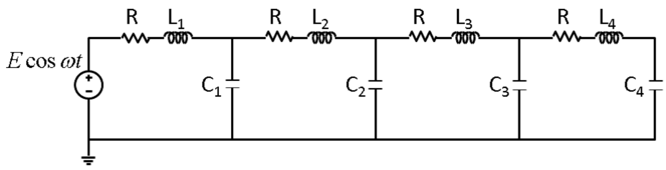

Fig. 1. LRC ladder circuit.

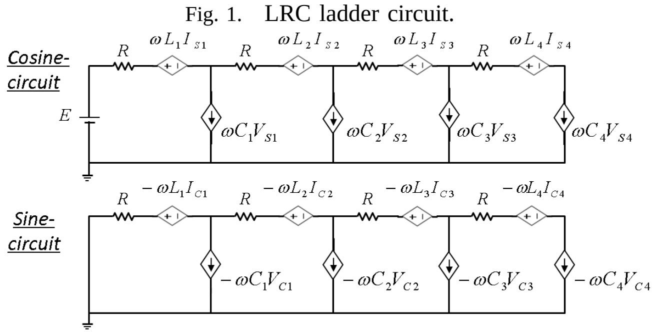

Fig. 2. Sine-cosine circuit for Fig.1.

$$v_C = V_{CS} \sin \omega t + V_{CC} \cos \omega t, \tag{1}$$

the current i_C is given by

$$i_C = C\frac{dv_C}{dt} = -\omega C V_{CC} \sin \omega t + \omega C V_{CS} \cos \omega t. \tag{2}$$

Thus, the coefficients of $\sin \omega t$ and $\cos \omega t$ are described by

$$I_{CS} = -\omega C V_{CC}, \quad I_{CC} = \omega C V_{CS}. \tag{3}$$

Namely, the capacitor is replaced by coupled voltage-controlled current sources in the sine-cosine transformation of the HB method. In the same way, let the current through an inductor L be

$$i_L = I_{LS} \sin \omega t + I_{LC} \cos \omega t. \tag{4}$$

Then, the voltage v_L is given by

$$v_L = L\frac{di_L}{dt} = -\omega L I_{LC} \sin \omega t + \omega L I_{LS} \cos \omega t. \tag{5}$$

Thus, the coefficients of $\sin \omega t$, $\cos \omega t$ are described by

$$V_{LS} = -\omega L I_{LC}, \quad V_{LC} = \omega L I_{LS}. \tag{6}$$

Namely, the inductor is replaced by coupled current-controlled voltage sources in the sine-cosine transformation.

As an example, Fig. 1 shows a LRC ladder circuit and Fig. 2 shows the corresponding sine-cosine circuits.

B. Fourier transformation circuit

In this study, we use the Fourier transformation circuit in order to realize the nonlinear characteristics of nonlinear capacitors.

Suppose the input and output waveforms as follows:

$$\begin{cases} i(t) = I_1 \cos \omega t + I_2 \sin \omega t \\ v(t) = V_1 \cos \omega t + V_2 \sin \omega t \end{cases}. \tag{7}$$

The characteristics of the electric current which flows through a capacitor can be indicated as

$$i = dq/dt = (\partial q/\partial v)(dv/dt). \tag{8}$$

From Eq. (8), the coefficients for electric charge $q(t)$ of $\sin \omega t$, $\cos \omega t$ are described by

$$q(t) = -\frac{1}{\omega} I_2 \cos \omega t + \frac{1}{\omega} I_1 \sin \omega t. \tag{9}$$

From this, input of Fourier transformation circuit model $i(t)$ is changed as

$$q(t) = Q_1 \cos \omega t + Q_2 \sin \omega t, \tag{10}$$

and from Eq. (8), the characteristics of a nonlinear capacitor is expressed with an equation using $q(t)$ and v as $v = G(q)$. We expand $G(q)$ to Fourier series, and obtain the coefficients of the voltages by using the trapezoidal formula as follows.

$$\begin{aligned} V_1 &= \frac{1}{\pi} \int_0^{2\pi} (G(q) \cos \omega t) d(\omega t) \\ &= \frac{1}{2K}(G_0 + G_{2K}) + \frac{1}{K}(G_1 \cos \theta_1 \\ &\quad + G_2 \cos \theta_2 + \ldots + G_{2K-1} \cos \theta_{2K-1}), \end{aligned} \tag{11}$$

$$\begin{aligned} V_2 &= \frac{1}{\pi} \int_0^{2\pi} (G(q) \sin \omega t) d(\omega t) \\ &= \frac{1}{K}(G_1 \sin \theta_1 + G_2 \sin \theta_2 + \ldots + G_{2K-1} \sin \theta_{2K-1}) \end{aligned} \tag{12}$$

where

$$\begin{aligned} \int_a^b G(q) d(\omega t) &= \frac{h}{2}(G_0 + G_{2K}) \\ &\quad + h(G_1 + G_2 + \ldots + G_{2K-1}), \end{aligned} \tag{13}$$

$$h = \frac{\pi}{K}, \tag{14}$$

$$G_i = G(q(t_i)), \tag{15}$$

$$\omega t_i = 0, \pi/K, ..., (2K-1)\pi/K, 2\pi. \tag{16}$$

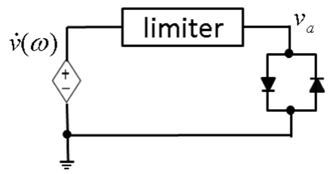

Fig. 3. Nonlinear limiter.

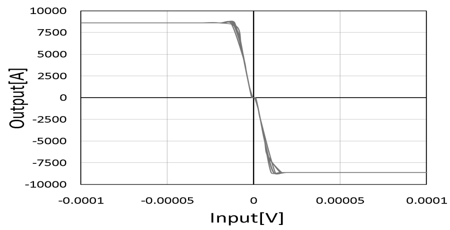

Fig. 4. Characteristics of nonlinear limiter.

III. Peak Search Algorithm

We search the peaks by using differentiator and nonlinear limiter. The frequencies ω at the peak voltages satisfy

$$\frac{d|v(\omega)|}{d\omega} = 0, \tag{17}$$

on the characteristics curve. Hence, $|v(\omega)|$ need to be firstly differentiated by a differentiator. In order to find the exact peak points, the output is limited and expanded with a nonlinear limiter, which consists of a limiter and nonlinear diode as shown in Fig. 3.

We suppose the characteristics of the limiter as follows.

$$v_a = \begin{cases} -V_{max} & : \quad \text{for } \; v_{in} < -V_L \\ k\, v_{in} & : \quad \text{for } \; -V_L \le v_{in} \le V_L \\ V_{max} & : \quad \text{for } \; V_L < v_{in} \end{cases}, \tag{18}$$

The output of the nonlinear limiter is given by

$$i_{out} = \begin{cases} I_s \exp(\lambda v_a) & : \quad \text{for } \; v_a > 0 \\ -I_s \exp(-\lambda v_a) & : \quad \text{for } \; v_a < 0 \end{cases}, \tag{19}$$

as shows in Fig. 4.

The expansion factor of k is large enough. In order to complicate an analysis around the peaks, we include nonlinear diode in nonlinear limiter. Thus, the transient analysis around the zero points is executed with a very small step size, and we can find out exact peak points.

IV. Trace for unstable region

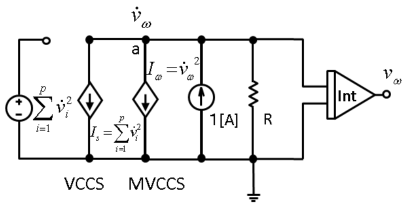

Fig. 5. STC (Solution trace circuit).

Since we set time as frequency, we can not analyze unstable region. In this section, we explain STC (solution trace circuit) for change a horizontal axis into a voltage v_ω from time (namely, frequency).

STC is based on the arc-length method [5][6]. Those voltages are differentiated with respect to the time t instead of the arc-length s by using differentiators. They are transformed to the corresponding voltage sources with current controlled voltage source (CCVS). Next, each voltage is squared and transformed to the current source. We have

$$I_s = \sum_{i=1}^{p} \left(\frac{dv_i}{dt} \right)^2 \tag{20}$$

as shown in Fig. 5. If we set the voltage of node a as $\dot{v}_\omega$, $I_\omega = \dot{v}_\omega^2$ can be obtained by multiplier and VCCS (MVCCS). Thus, the additional constant current source in Fig. 5 realize to satisfying the arc-length method by Eq. (20). Then, the node voltage $\dot{v}_\omega$ is integrated to obtain v_ω. Note that R in Fig. 5 is a very large resistance used only to avoid the L-J cut-set.

V. Illustrative Example

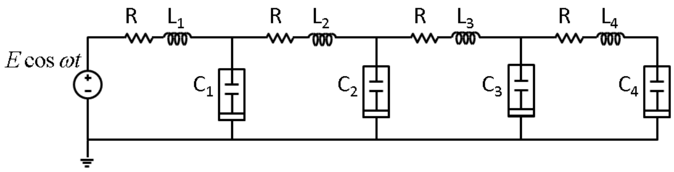

Fig. 6. LRC ladder circuit including nonlinear capacitor.

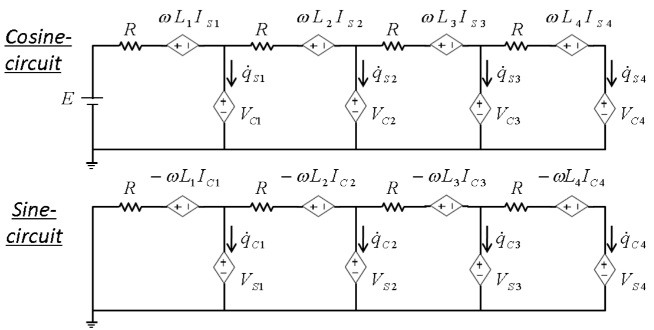

Fig. 7. Sine-cosine circuit for Fig. 7.

As an example we consider the LRC ladder circuit including nonlinear capacitors as Fig. 6. Figure 7 is the sine-cosine circuit for Fig. 6. In Fig. 6, $L_1 = L_2 = L_3 = L_4 = 0.1[H]$, and the nonlinear characteristics of C_1, C_2, C_3, C_4 are the same. We set $K = 10$ and the characteristics of the nonlinear capacitors are $G(q) = q + 0.8q^3$. The voltages of the CCVS in Fig. 7 are inputted to the STC. We set v_ω in Fig. 5 as ω for the frequency characteristics.

The simulated results are shown in Fig. 8 for the cases of $R = 0.01[\Omega]$, respectively. The horizontal axis is ω and the vertical axis is the voltage through the nonlinear capacitor C_1. We name the peaks of the curves as peak1, peak2, peak3 and peak4 from the left. The peaks of the curves become inclined as reducing the resistance (this corresponds to increase the effect of the nonlinearity). Although the unstable region appears in the case of $R = 0.01[\Omega]$, we could trace the curve successfully as shown in Fig. 8. We can also notice that the step size around the peak becomes smaller by the effect of the nonlinear limiter. We should mention that each frequency characteristics curve could be obtained by a single run of the transient analysis of SPICE.

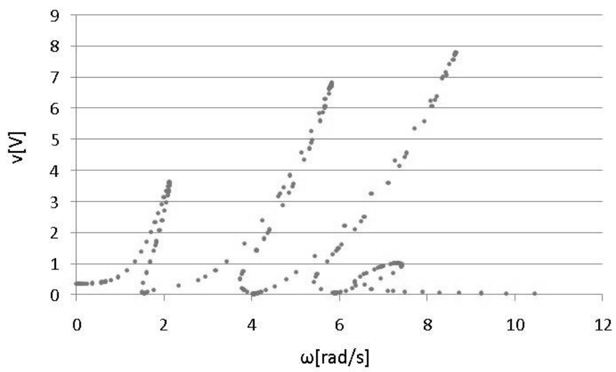

Fig. 8. Plot of frequency characteristics for R=0.01[Ω].

VI. Conclusions

We have proposed a SPICE-oriented algorithm to analyze frequency characteristics with unstable region and to find the peaks of the frequency characteristics. By combining the sine-cosine circuits, the Fourier transformation circuit, the nonlinear limiter and the solution tracing circuit, the frequency characteristics curve can be obtained even if the curve has unstable region. The simulation results of the LRC ladder circuit with nonlinear capacitors showed the efficiency of the proposed method.

The analysis of printed circuit boards using our proposed method is our future research.

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