Bifurcation and Synchronization in Coupled Parametrically Forced Logistic Maps

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***Abstract*— In this study, a parametrically forced logistic map that a parameter of the logistic map are forced into periodic varying is suggested. Unique bifurcations from period to chaos are observed in the map. Then, synchronization phenomena in globally coupled system of the parametrically forced logistic map are investigated. When the number of coupling is three, various synchronization phenomena are observed by choosing a coupling intensity. The synchronization phenomena fall into three general categories, which are asynchronous, self-switching of synchronization, synchronization of two among the three maps and synchronization of all the maps. Further more, relationship between sojourn time and the coupling intensity in the self-switching of synchronization is investigated. The sojourn time increase exponentially with the coupling intensity.**

I. Introduction

Synchronization is one of the fundamental phenomena in nature, and one of typical nonlinear phenomena. Therefore, studies on synchronization phenomena of coupled systems are extensively carried out in various fields, physics [1], biology [2], engineering and so on. However, the issues that should be investigated for synchronization remain in existence in spite of many researching. In particular, it is necessary to investigate synchronization phenomena in special conditions. There is parametric excitation that an amplitude of oscillation is increased by periodic varying of a parameter in some system. Parametric excitation circuit is one of resonant circuits, and it is important to investigate various nonlinear phenomena for future engineering applications. In a simple oscillator including parametric excitation, Ref. [3] reports that the almost periodic oscillation occurs if nonlinear inductor has saturation characteristic. Additionally the occurrence of chaos is referred in Refs. [4] and [5].

By the way, spatiotemporal chaotic phenomena that spatially-extended systems indicate temporal and spatio complex patterns have been studied. Behavior generated in coupled system of chaotic one-dimensional map is investigated in Refs. [6]-[8] In particular, Coupled Map Lattice (CML) and Globally Coupled Map (GCM) are well known as mathematical models in discrete-time system. Various kinds of dynamics are observed in their systems. The research into CML and GCM are important for not only modeling of multiple degree of freedom nonlinear systems but also suggestion to biological networks and engineering applications. In the past we have investigated effects of parametric excitation in coupled van der Pol oscillators [9]. In this study, for more detailed investigation of effect of the parametric excitation on synchronization, we focus on a globally coupled system of simple one-dimensional maps. A typical scheme for global coupling is given by

$$x_i(t+1) = (1-\varepsilon)f[x_i(t)] + \frac{\varepsilon}{N}\sum_{j=1}^{N} f[x_j(t)] \qquad (i = 1, 2, \cdots, N,) \tag{1}$$

where ε is the coupling intensity. The globally coupled maps are a scheme that an average number of all the maps affect each of the maps, and similar to the system that we have studied using van der Pol oscillators. Hence, we investigate synchronization phenomena in a globally coupled system of one-dimensional maps which are forced by periodic parameter change. The one-dimensional map used in this study is a logistic map, since the map can be described by a simple discrete equation. The logistic map is a polynomial mapping, often cited as an archetypal example of how complex chaotic behavior can be arisen from very simple nonlinear dynamical equations. Mathematically, the logistic map is written as

$$x(t+1) = \alpha x(t)(1 - x(t)). \tag{2}$$

Firstly, we describe behaviors and bifurcations of the parametrically forced logistic map. Next, we investigate synchronization phenomena in the globally coupled parametrically forced logistic maps.

II. Parametrically forced logistic map

A parametrically forced logistic map used in this study is described as:

$$x(t+1) = \alpha_f(t)x(t)(1 - x(t)), \tag{3}$$

and

$$\alpha_f(t) = \begin{cases} \alpha_1, & n(\tau_1+\tau_2) < t \le n(\tau_1+\tau_2)+\tau_1 \\ \alpha_2, & n(\tau_1+\tau_2)+\tau_1 < t \le (n+1)(\tau_1+\tau_2) \\ & (n = 1, 2, \ldots) \end{cases}, \tag{4}$$

where $\alpha_f(t)$ is a term of the parametric force and time-varying. The parametric force operation can be described as follows: in the time interval $n(\tau_1+\tau_2) < t \le n(\tau_1+\tau_2)+\tau_1$, the system is driven by a parameter α_1 during the duration τ_1;

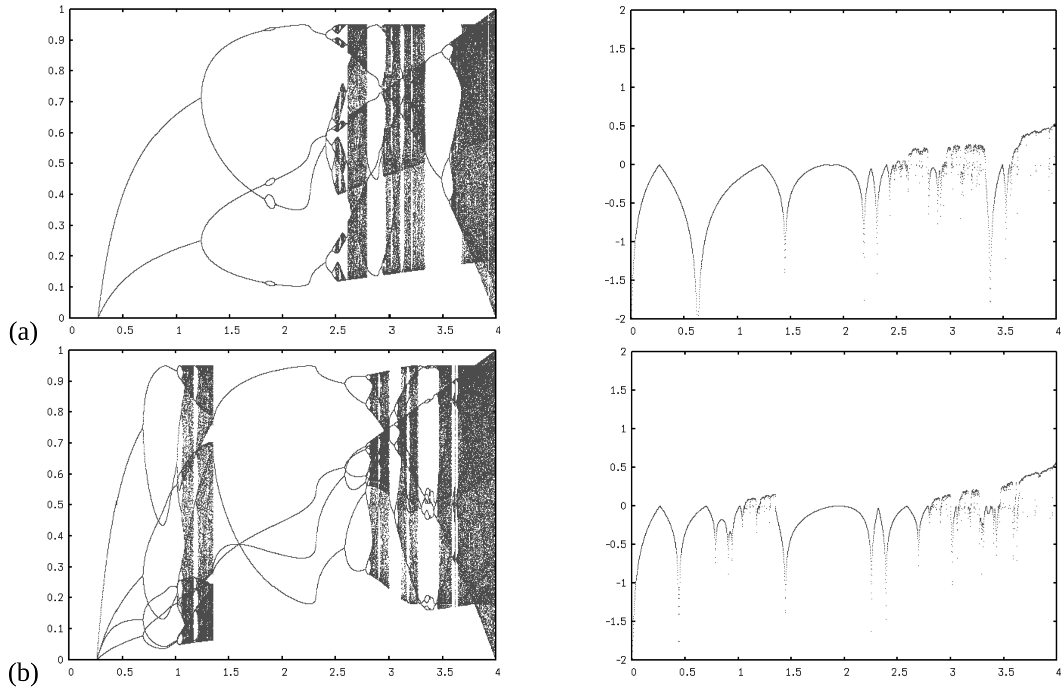

Fig. 1. One-parameter bifurcation diagrams (left) and the Lyapunov exponents (right) for $\alpha_1 = 3.8$. Horizontal axis: α_2. (a) $\tau = 1$. (b) $\tau = 2$.

while in the interval $n(\tau_1 + \tau_2) + \tau_1 < t \leq (n+1)(\tau_1 + \tau_2)$, the system is driven by a parameter α_2 during the duration τ_2. Namely, in this system, two kinds of parameters are replaced alternately by the number of updates. Then, a parameter giving a periodic solution and a parameter giving another periodic solution can be combined. Of course, other combinations, for instance two parameters giving a periodic solution and a chaotic solution or two parameters giving two kinds of chaotic solutions, are possible. In this study, we assume $\tau_1 = \tau_2 = \tau$ for simplicity.

III. Bifurcation

Before investigating synchronization phenomena in the globally coupled parametrically forced logistic maps, it is necessary to investigate behavior and bifurcation of one uncoupled parametrically forced logistic map. Figure 1 shows computer calculated results. Fixing a parameter $\alpha_1 = 3.8$ and $\tau = 1$ and varying a parameter α_2, the one-parameter bifurcation diagram and the Lyapunov exponents are obtained as shown in Fig. 1(a). Figure 1(b) shows the bifurcation diagram and the Lyapunov exponents for $\tau = 2$. The different types of bifurcation diagrams are obtained. From these figures, observations of periodic and chaotic attractors are confirmed.

Figure 2 show some examples of the return maps of the parametrically forced logistic maps. For the original logistic map, two-periodic solution is observed for $\alpha = 3.0$. While, three-periodic solution is observed for $\alpha = 3.8$. These two solutions are periodic, whereas in the logistic map involving parametric force, a solution is chaotic as shown in Fig. 2(a) when the parameters α_1 and α_2 are set 3.0 and 3.8. Namely,

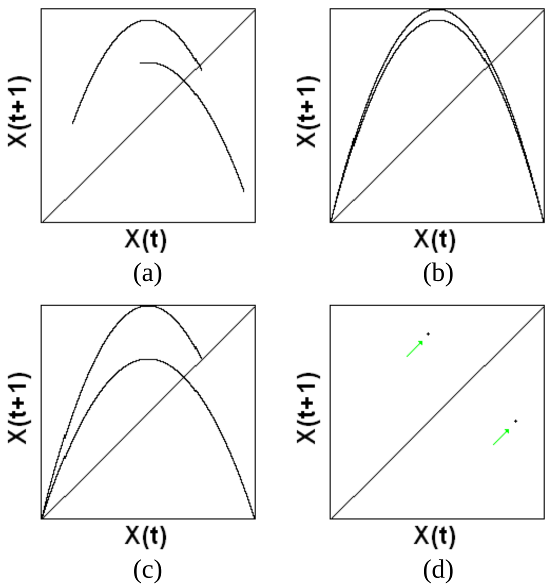

Fig. 2. Return maps of parametrically forced logistic maps for $\tau = 1$. (a) $\alpha_1 = 3.0$ and $\alpha_2 = 3.8$. (b) $\alpha_1 = 3.8$ and $\alpha_2 = 4.0$. (c) $\alpha_1 = 3.0$ and $\alpha_2 = 4.0$. (d) $\alpha_1 = 3.5$ and $\alpha_2 = 4.0$.

chaotic solution can be observed in the combination of two parameters that generate two kinds of periodic solutions.

IV. Synchronization

Synchronization phenomena generated in the globally coupled logistic map involving parametric force are investigated for one control parameter ε which is a coupling intensity when

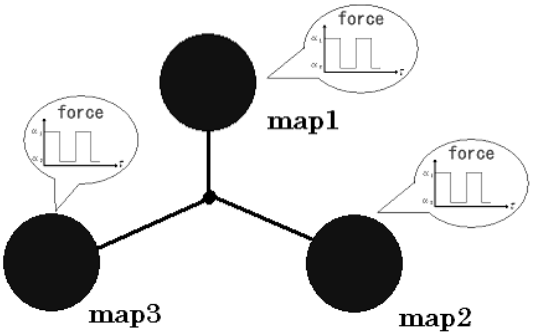

Fig. 3. Conceptual structure of three coupled parametrically forced logistic maps.

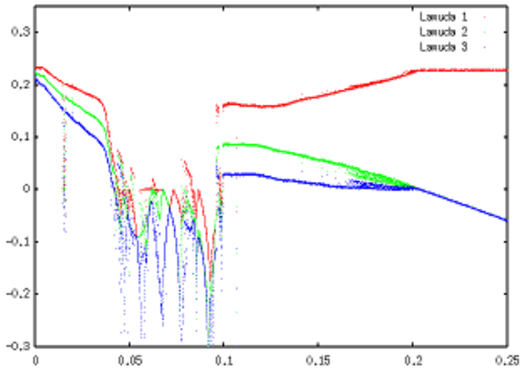

Fig. 4. Lyapunov exponents in globally coupled parametrically forced logistic maps for $\alpha_1 = 3.0$, $\alpha_2 = 3.8$ and $\tau = 1$. Horizontal axis: ε.

the number of coupling is three. Figure 3 shows a conceptual structure of teh three coupled parametrically forced logistic maps. In the following computer calculations, the parameters are fixed as $\alpha_1 = 3.0$, $\alpha_2 = 3.8$ and $\tau = 1$.

The three-dimensional Lyapunov exponents obtained at $\alpha_1 = 3.8$, $\alpha_2 = 4.0$ and $\tau = 1$ are shown in Fig. 4. In Fig. 4, λ_1 is maximal Lyapunov exponent. Figure 5 shows examples of synchronization phenomena. In Fig. 5, upper figures show the return maps and lower figures show the phase differences between the maps.

First, when the coupling parameter ε is small, three lyapunov exponents are almost same and the maps are almost asynchronous as shown in Fig. 5(a). Increasing the coupling intensity over 0.0400, the lyapunov exponents decrease rapidly. Then a self-switching phenomenon of synchronization is observed. As increasing the coupling intensity, λ_1 becomes negative and attractors of the maps become periodic. Then, two of the three maps are synchronized as shown in Fig. 5(b). Moreover, as increasing the coupling intensity over 0.100, all the λ become positive and the attractors become chaotic as shown in Fig. 5(c). Although all the maps become to be asynchronous. After that, λ_3 comes close to zero. Then two of the three maps are synchronized as shown in Fig. 5(d). In the figure, all attractors behave chaotic, and map 1 and map 3 are synchronized. While, the shapes of the return maps are

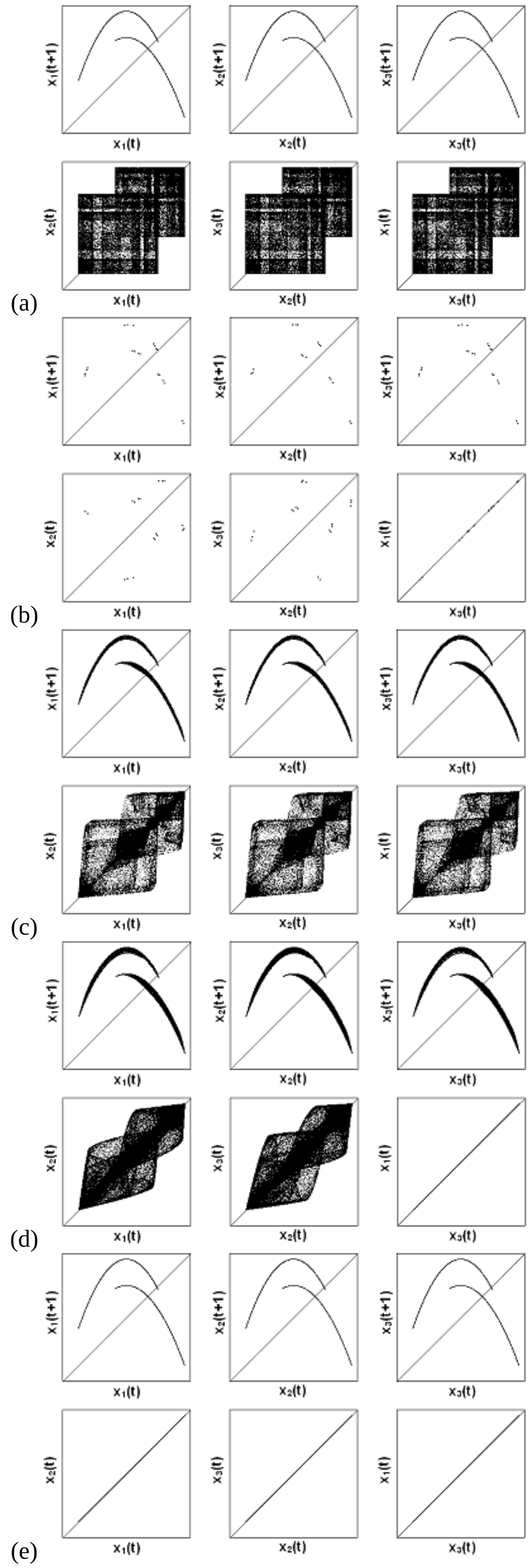

Fig. 5. Synchronization of three chaos. $\alpha_1 = 3.0$, $\alpha_2 = 3.8$ and $\tau = 1$. (a) $\varepsilon = 0.000$. (b) $\varepsilon = 0.085$. (c) $\varepsilon = 0.100$. (d) $\varepsilon = 0.170$. (e) $\varepsilon = 0.200$.

different from that of the uncoupled map. Finally, when the coupling intensity ε is over 0.2, λ_2 and λ_3 become negative. Then, all the maps are synchronized as shown in Fig. 5(e). In the figure, all attractors are chaotic and have the same shapes as a attractors of the uncoupled map.

V. Self-switching phenomenon

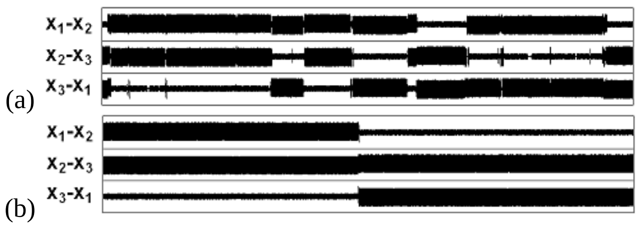

Fig. 6. Time series of differences between two maps. $\alpha_1 = 3.0$, $\alpha_2 = 3.8$ and $\tau = 1$. (a) $\varepsilon = 0.0400$. (b) $\varepsilon = 0.0415$.

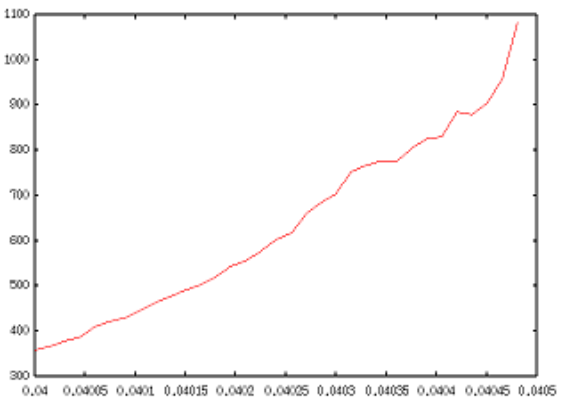

Fig. 7. Sojourn time of the self-switching. $\alpha_1 = 3.0$, $\alpha_2 = 3.8$ and $\tau = 1$. Hertical axis: ε, vertical axis: the mean value of the sojourn time.

The self-switching phenomenon of synchronization is observed when the ε is set around 0.04. The phenomenon is that two among the three maps are synchronized and the combination which maps are coupled changes with time. Figure 6 shows time series of differences of $x(t)$ between two maps. Areas where the amplitudes of the time series are small correspond to in-phase synchronization in the figure. In Fig. 6(a), firstly, map 1 and map 2 are synchronized. However, after a time, the synchronous state breaks up and map 1 and map 3 are synchronized. As seen above, the synchronous states switch with time in sequence. Additionally, a sojourn time of the self-switching is related to ε. The sojourn time is short when ε is small, whereas the sojourn time is long when ε is big as shown Figs. 6(a) and (b). Figure 7 shows sojourn time of the self-switching between $\varepsilon = 0.04$ and $\varepsilon = 0.0405$. The sojourn time increases exponentially with ε.

VI. Conclusion

In this study, parametrically forced logistic map was suggested. Unique bifurcations from period to chaos were observed. Then, synchronization phenomena in globally coupled system of the parametrically forced logistic map were investigated. For the number of coupling is three, various synchronization phenomena are observed by choosing a coupling intensity. The synchronization phenomena fall into three general categories, which are asynchronous, self-switching of synchronization, synchronization of two among the three maps and synchronization of all the maps. Further more, relationship between sojourn time and the coupling intensity in the self-switching of synchronization was investigated. The sojourn time increase exponentially with the coupling intensity.

Acknowledgment

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References

[1] I. Belykh, M. Hasler, M. Lauret and H. Nijmeijer, “Synchronization and graph topology,” Int. J. Bifurcation and Chaos, vol.15, no.11, pp.3423-3433, Nov. 2005.

[2] J. Cosp, J. Madrenas, E. Alarcon, E. Vidal and G. Villar, “Synchronization of nonlinear electronic oscillators for neural computation,” IEEE Trans. Neural Networks, vol.15, no.5, pp.1315-1327, Sep. 2004.

[3] C. Hayashi, “Nonlinear Oscillations in Physical Systems,” Chap. 11, McGraw-Hill, New York (1964).

[4] C. Hayashi, M. Abe, K. Oshima and H. Kawakami, “The Method of Mapping As Applied To the Solution for Certain Types of Nonlinear Differential Equations,” Proc. of the Ninth International Conference on Nonlinear Oscillations, pp. 1-8, 1981.

[5] M. Inoue, “A Method of Analysis for the Bifurcation of the Almost Periodic Oscillation and the Generation of Chaos in a Parametric Excitation Circuit,” Trans. of IEICE, vol. J68-A, no. 7, pp. 621-626, 1985.

[6] K. Kaneko, “Spatiotemporal intermittency in couppled map lattice,” Prog. Theor. Phys., vol.74, no.5, pp.1033-1044, Nov. 1985.

[7] K. Kaneko, “Pattern dynamics in spatiotemporal chaos,” Physica D, vol.34, nos.1-2, pp.1-41, Jan. 1989.

[8] K. Kaneko, “Clustering, coding, switching, hierarchical ordering, and control in a network of chaotic elements,” Physica D, vol.41, nos.2, pp.137-172, March. 1990.

[9] H. Kumeno and Y. Nishio, “Synchronization Phenomena in Coupled Parametrically Excited van der Pol Oscillators,” Proceedings of International Symposium on Nonlinear Theory and its Applications (NOLTA’ 08), pp. 128- 131, Sep. 2008