

# Synchronization Phenomena in Coupled Simultaneous Oscillators

Chikayasu HIGASHI Yoshifumi NISHIO  
(Tokushima University)

## 1. Introduction

There have been many investigations of the mutual synchronization of oscillators. Because mutual coupled oscillators has various phase states. Schaffner confirmed that an oscillator with two degree of freedom can oscillate simultaneously at two different frequencies when the nonlinear characteristics is fifth-power [1]. In this study, we investigate a coupled system of simultaneous oscillators and observe synchronization phenomena.

## 2. Circuit Model

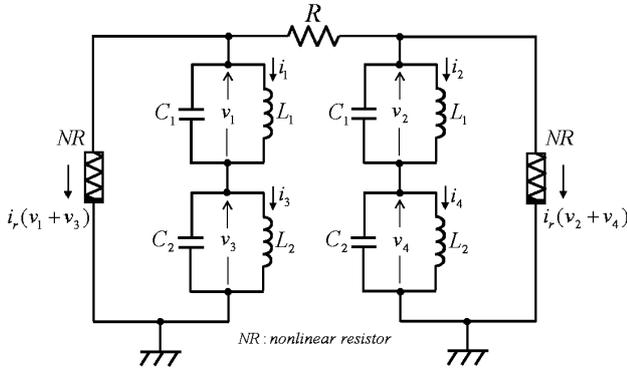


Figure 1: Circuit model.

The circuit model is shown in Fig. 1. The circuit equations are described as follows,

$$\begin{cases} v_1 = L_1 \frac{di_1}{dt}, v_2 = L_1 \frac{di_2}{dt}, v_3 = L_2 \frac{di_3}{dt}, v_4 = L_2 \frac{di_4}{dt} \\ C_1 \frac{dv_1}{dt} = -i_1 - \frac{1}{R}(v_1 + v_3 - v_2 - v_4) - i_r(v_1 + v_3) \\ C_1 \frac{dv_2}{dt} = -i_2 + \frac{1}{R}(v_1 + v_3 - v_2 - v_4) - i_r(v_2 + v_4) \\ C_2 \frac{dv_3}{dt} = -i_3 - i_r(v_1 + v_3) \\ C_2 \frac{dv_4}{dt} = -i_4 - i_r(v_2 + v_4) \end{cases} \quad (1)$$

where  $i_r(v_k)$  indicates the  $v-i$  characteristics of the nonlinear resistors and we consider the two cases of third-power and fifth-power nonlinear characteristics as follows.

$$i_r(v) = \begin{cases} -g_1 v + g_3 v^3 & (g_1, g_3 > 0) \\ \text{(third - power} \\ \text{nonlinear characteristics)} \\ g_1 v - g_3 v^3 + g_5 v^5 & (g_1, g_3, g_5 > 0) \\ \text{(fifth - power} \\ \text{nonlinear characteristics)} \end{cases} \quad (2)$$

## 3. Synchronization Phenomena

As showing in Fig. 2, the oscillators with the third-power nonlinear characteristics cannot oscillate simultaneously at

two different frequencies. On the other hand, as showing in Fig. 3, the oscillators with the fifth-power nonlinear characteristics can oscillate.

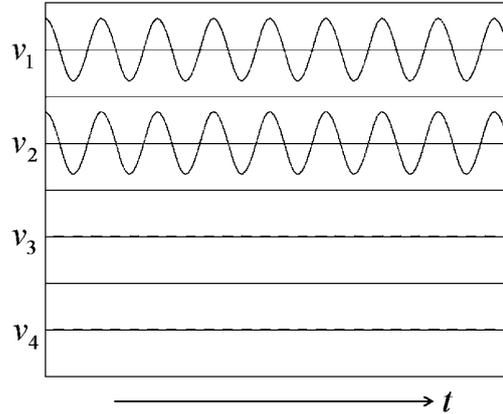


Figure 2: Time waveform of in-phase oscillation for  $L_1/L_2=2.0$  and  $C_1/C_2=1.0$ . (third-power nonlinear characteristics).

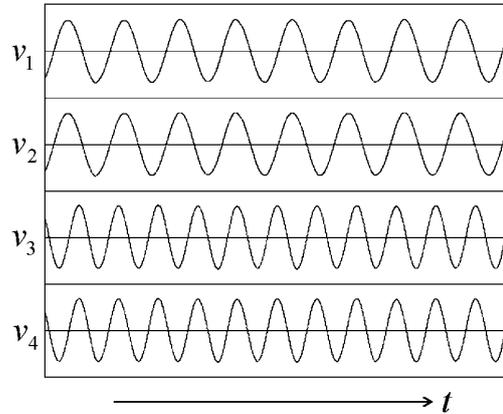


Figure 3: Time waveform of simultaneous oscillation for  $L_1/L_2=2.0$ ,  $C_1/C_2=1.0$ . (fifth-power nonlinear characteristics).

## 4. Conclusions

In this article, we presented synchronization phenomena in coupled simultaneous oscillators. The system with the fifth-power nonlinear characteristics gives more interesting synchronization phenomena than that with the third-power nonlinear characteristics. Our future work is to investigate these synchronization phenomena in more detail.

## References

[1] Schaffner J. : "Simultaneous oscillations in oscillators", IRE Trans. on Circuit Theory, 1, pp. 2-8 (June 1954).