1. Introduction

Recently, the synchronization phenomena in coupled chaotic oscillators are studied by many researchers. Because the synchronization phenomena have caused very interesting phenomena. However, a lot of synchronization phenomena of coupled chaotic oscillators have not been solved yet and we have to investigate them.

In this study, we propose chaotic oscillators coupled via switch. We investigate synchronization phenomena in coupled auto gain controlled oscillators containing time delay. We observe several interesting synchronization phenomena.

2. Circuit model

![Circuit model](image)

The circuit model used in this study is shown in Fig. 1. In the circuit, two chaotic oscillators which are introduced in [1] are coupled by periodically changing switch SW3. The periodically changing switch connects alternately with one subcircuit and the other with a fixed time interval (the switching frequency is $2\pi \gamma_0$). Two comparators of the subcircuits obtain information of the voltage from the subcircuit connected via SW3 and compare the voltage of the subcircuit with a given threshold $V_{th}$. The periodically changing switch can be controlled by the following sinusoidal function.

$$k = \sin \gamma_0 t.$$ (1)

The periodically changing switch connects to the left subcircuit in Fig. 1 when $k$ is positive, while to the right subcircuit in Fig. 2 when negative. The voltage of the subcircuit connected via SW3 controls SW1 and SW2 at the same time. If the voltage of the subcircuit is lower than the threshold $V_{th}$, SW1 and SW2 are connected to the negative resistor, while if the voltage of the subcircuit rises above $V_{th}$, SW1 and SW2 are connected to the positive resistor after the given delay time during the time interval while the voltage of the subcircuit is higher than $V_{th}$.

By using the following normalized variables and parameters,

$$i_i = \sqrt{C/L}V_{th} x_i, \quad v_i = V_{th} y_i, \quad t = \sqrt{LC} r, \quad \gamma_0 = \gamma/\sqrt{LC}, \quad g\sqrt{L/C} = 2\alpha, \quad G\sqrt{L/C} = 2\beta,$$

the normalized circuit equations are given as:

$$\begin{align*}
\dot{x}_i &= y_i \\
\dot{y}_i &= -x_i + 2\alpha y_i
\end{align*}$$ (3)

$$\begin{align*}
\dot{x}_i &= y_i \\
\dot{y}_i &= -x_i - 2\beta y_i.
\end{align*}$$ (4)

When SW1 and SW2 are connected to the negative resistor, the circuit is governed by Eq. (3), while when SW1 and SW2 are connected to the positive resistor, the circuit is governed by Eq. (4).

3. Synchronization phenomena

We carry out computer calculations for coupled auto gain controlled oscillators containing time delay. We observe coexistence phenomena of in-phase synchronization and opposite-phase synchronization. Figure 2 shows examples of the observed attractors and phase differences at the parameters $\alpha = 0.015$, $\beta = 0.89$ and $\gamma = 0.02911$. The two subcircuits are synchronized at in-phase in Fig. 2(a), while the two subcircuits are synchronized at opposite-phase in Fig. 2(b). These states are observed for the same parameters, however initial values are different. Thus, the in-phase synchronization and the opposite-phase synchronization coexist.

![Attractors and phase differences](image)

4. Conclusions

In this study, we have proposed coupled auto gain controlled oscillators containing time delay. In this circuit, we observed coexistence phenomena of in-phase synchronization and opposite-phase synchronization. Our future research is to make clear the mechanism of the generation of the coexistence and to carry out circuit experiments.

References