18-14

Synchronization Phenomena in Coupled Cubic Maps

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1. Introduction

Synchronization phenomena in complex systems are very good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on chaos synchronization in coupled chaotic circuits are extensively carried out in various fields [1]. On the other hand, chaotic maps are generally used for several approaches to investigate chaotic phenomena on coupled chaotic systems. Especially, a logistic map, a circle map, a tent map, a cubic map are well known and popular. In this study, we investigate synchronization phenomena in coupled cubic maps.

2. Cubic map

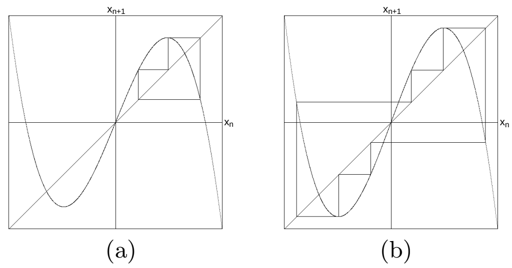

Figure 1: Cubic map. (a) $\alpha = -3.451$. (b) $\alpha = -3.70$.

$$x_{(n+1)} = \alpha x_n^3 - (\alpha+1)x_n \tag{1}$$

Figure 1 shows the cubic map Eq. (1) where n is an iteration, α is a parameter which determines the chaotic feature. We can easily confirm that it generates various periodical solutions and chaos. Figure 1(a) shows asymmetric 3 periodic solution and Fig. 1(b) shows symmetric 6 periodic solution.

3. Coupled cubic maps

In this study, we consider two coupled cubic maps. The followings are the equations of coupled cubic maps where ϵ is a coupling parameter.

$$\begin{cases} f(x_{(n)}) = \alpha x_n^3 - (\alpha+1)x_n \\ x_{1(n+1)} = (1-\epsilon)f(x_{1(n)}) + \epsilon(f(x_{2(n)})) \\ x_{2(n+1)} = (1-\epsilon)f(x_{2(n)}) + \epsilon(f(x_{1(n)})) \end{cases} \tag{2}$$

Figure 2 shows examples of the computer simulated results of Eq. (2). Figure 2(a) shows the timewaveform obtained with the parameter near that giving the periodic solution in Fig. 1(a). In the same way, Fig. 2(b) shows the timewaveform obtained with the parameter near that giving the periodic solution in Fig. 1(b). From the coupled cubic maps, we can observe interesting state synchronization phenomena. The two maps synchronized in anti-phase in Fig. 2(a) and synchronized in in-phase in Fig. 2(b). We could confirm that the two maps synchronize only for the parameters near that giving periodic solutions. However, we do not understand the relationship between the obtainable synchronization states and symmetry property of the original solutions.

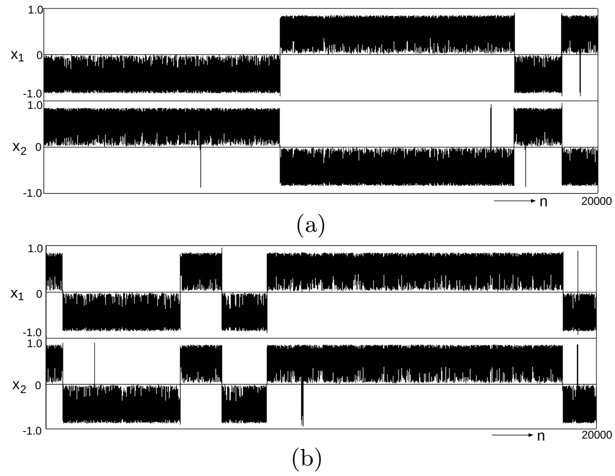

Figure 2: Timewaveform of coupled cubic maps. (a) $\alpha = -3.45043$, and $\epsilon = -0.0477$. (b) $\alpha = -3.69964$, and $\epsilon = 0.0508$.

Figure 3 shows how the sojourn times between the state transitions change as the coupling parameter ϵ changes. The horizontal axis is coupling parameter ϵ and the vertical axis is the average lengths of the iterations n. From this figure, we can see that the sojourn time between the transitions becomes longer as increasing the coupling parameter ϵ.

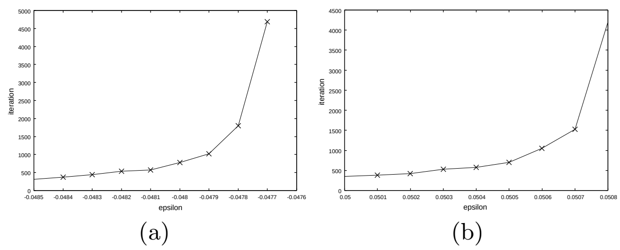

Figure 3: Sojourn time between state transitions. (a) $\alpha = -3.45043$. (b) $\alpha = -3.69964$.

4. Conclusions

In this study, we have investigated the synchronization phenomena of two coupled cubic maps. We could observe interesting synchronization phenomenon. We confirmed that the two maps synchronized for the parameters near that giving periodic solutions. And, the sojourn time between the transitions became longer as increasing the coupling parameter. Clarifying the mechanism is our future work.

References

[1] P. Ashwin, J. Buescu and I. Stewart, "Bubbling of Attractors and Synchronization of Chaotic Oscillators," Phys. Lett. A, 193, pp. 126-139, 1994.