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Synchronization Phenomena in Globally Coupled Parametrically Forced Logistic Maps

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1. Introduction

Synchronization is one of the fundamental phenomena in nature, and one of typical nonlinear phenomena. Therefore, studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields.

In the past we have investigated effects of parametric excitation for synchronization [1]. In this study, for additional investigation of the effect of parametric excitation on synchronization, we focus on a globally coupled system of one-dimensional map. A typical scheme for global coupling is given by

$$\begin{aligned} &x_i(t+1) = (1-\varepsilon)f[x_i(t)] + \tfrac{\varepsilon}{N}\sum_{j=1}^{N} f[x_j(t)] \\ &i = 1, 2, \cdots, N, \end{aligned} \tag{1}$$

where $\varepsilon \in [0, 1]$ is the coupling intensity. The globally coupled map is a scheme that an average number of all the maps affect each of the map, and similar to the system that we have studied. Hence we investigate synchronization phenomena in a globally coupled system of one-dimensional maps which are forced into periodic parameter change. The one-dimensional map used in this study is a logistic map since the map can be described by a simple discrete equation.

Firstly, we describe behaviors and bifurcations of the parametrically forced logistic map. Next, we investigate synchronization phenomema in the globally coupled parametrically forced logistic maps.

2. Parametrically forced logistic map

A parametrically forced logistic map used in this study is described as:

$$x_i(t+1) = \alpha_f(t) x_i(t)(1 - x_i(t)), \tag{2}$$

and

$$\alpha_f(t) = \begin{cases} \alpha_1, & n(\tau_1+\tau_2) < t \le n(\tau_1+\tau_2)+\tau_1 \\ \alpha_2, & n(\tau_1+\tau_2)+\tau_1 < t \le (n+1)(\tau_1+\tau_2) \\ & (n = 1, 2, ...) \end{cases}, \tag{3}$$

where $\alpha_f(t)$ is a term of the parametric force and time-varying. The parametric force operation can be described as follows: in the time interval $n(\tau_1+\tau_2) < t \le n(\tau_1+\tau_2)+\tau_1$, the system is driven by parameter α_2; while in the interval $n(\tau_1+\tau_2)+\tau_1 < t \le (n+1)(\tau_1+\tau_2)$, the system is driven by parameter α_2 during the duration τ_2. In this study, we assume $\tau_1 = \tau_2 = \tau$ for simplify. One parameter bifurcation diagram that α_1 and τ are fixed and α_2 is varying is shown in Fig. 1. Periodic, quasi-periodic and chaotic attractors are confirmed. Figure 2 shows an example of chaotic return maps at $\alpha_2 = 4.0$.

3. Synchronization

We carry out computer calculations for the globally coupled three parametrically forced logistic maps and investigate synchronization phenomena of the coupled maps. In this case, various kinds of synchronization phenomena are

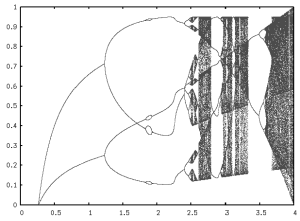


Figure 1: One-parameter bifurcation diagram. Vertical axis: $x_i(t)$. Horizontal axis: α_2. $\alpha_1 = 3.8$ and $\tau = 1$.

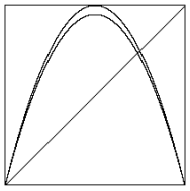
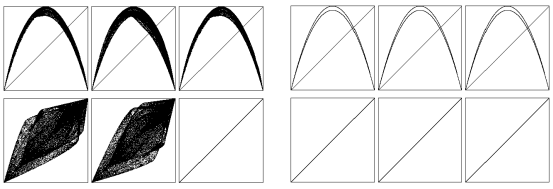


Figure 2: Return map of parametrically forced logistic map. $\alpha_1 = 3.8$, $\alpha_2 = 4.0$ and $\tau = 1$.

confirmed with increasing coupling intensity ε. Figure 3 shows examples of synchronization phenomena. In Fig. 3, upper figures show return maps and lower figures show phase differences between the maps. First, when coupling parameter ε is small, the maps are almost asynchronous. With increasing coupling intensity ε, the synchronization states are changed as: asynchronous → synchronizing of two of the three maps which generate periodic attractors → synchronizing of two of the three maps which generate chaotic attractors (see Fig. 3(a)) → synchronizing of all the maps which generate chaotic attractors (see Fig. 3(b)).



(a) (b)

Figure 3: Return maps and phase differences. $\alpha_1 = 3.8$, $\alpha_2 = 4.0$ and $\tau = 1$. (a) $\varepsilon = 0.32$. (b) $\varepsilon = 0.41$.

4. Conclusions

In this study, we investigated synchronization phenomena of globally coupled three parametrically forced logistic maps. By carrying out computer calculations for the system, we confirmed various kinds of synchronization phenomena. The synchronization states are changed with increasing coupling intensity as: asynchronous → synchronizing of two of the three maps → synchronizing of all the maps.

Reference

[1] H. Kumeno and Y. Nishio, "Synchronization Phenomena in Coupled Parametrically Excited van der Pol Oscillators," Proceedings of International Symposium on Nonlinear Theory and its Applications (NOLTA '08), pp. 128-131, Sep. 2008