TC4-5

CHAOS SYNCHRONIZATION BY CROSSTALK OF TRANSMISSION LINES

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Abstract

In this study, two Chua's circuits with lossless transmission lines placed in parallel are investigated. By computer simulations, it is confirmed that the two Chua's circuits can be synchronized by the effect of the crosstalk. Further, the existence of the solution on the in-phase synchronization can be explained theoretically.

Keywords: chaos synchronization, Chua's circuit, cross talk, transmission line.

1. Introduction

Chua's circuit is one of the simplest autonomous chaotic circuits. Chua's circuit consists of a resistor, a capacitor, a nonlinear resistor and an LC resonator as shown in Fig. 1. The nonlinear resistor in Chua's circuit has a piecewise linear $v-i$ characteristics shown in Fig. 2.

Chua's circuit whose LC resonator is replaced by a transmission line (Fig. 3) is referred to as the "time-delayed Chua's circuit." [1−4] Because the time-delayed Chua's circuit with $C_0 = 0$ is governed by a one-dimensional difference equation, detailed analysis using the corresponding one-dimensional map is possible [1,2]. However, in that system we can not observe the same chaotic phenomena as those observed from the original Chua's circuit, e.g., Double Scroll attractors. Hosny et al. have proposed an analytical method for solving nonlinear lumped circuits with a lossless transmission line. As an example, chaotic behavior of the time-delayed Chua's circuit is analyzed for the case of $C_0 \neq 0$ [3]. Kawata and the author also investigated the same circuit with more general transmission line [4].

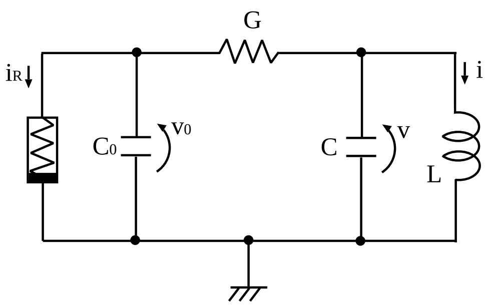


Fig. 1: Chua's circuit.

In this study, two Chua's circuits with lossless transmission lines placed in parallel are considered. Transmission lines placed in parallel cause the crosstalk phenomena. Crosstalk phenomena appear in long powerlines or very high speed VLSI and its effect is normally not preferable. Nakaaji and the author have already confirmed that two Chua's circuits

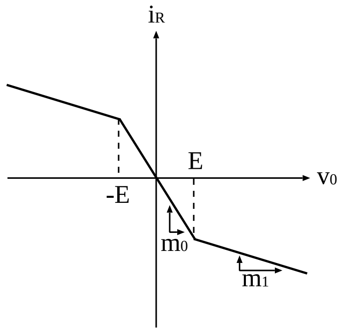


Fig. 2: $v-i$ characteristics of nonlinear resistor.

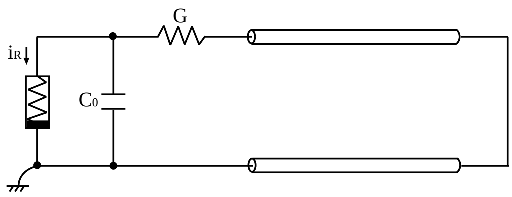


Fig. 3: Time-delayed Chua's circuit

with transmission lines could be synchronized by the effect of the crosstalk. [5] In this study, the synchronization of two Chua's circuits linked by the crosstalk is analyzed in more detail, especially, paying the attentions to the synchronization plane.

2. Chua's Circuits with Transmission Lines Placed with Same Direction

First, two Chua's circuits with transmission lines are considered when they are placed in parallel as shown in Fig. 4. Crosstalk of the transmission lines can be expressed by coupling capacitors C_M and mutual inductors L_M after replacing the transmission lines by lumped ladder LC circuits as shown in Fig. 5.

The circuit equations governing the circuit model in Fig. 5 can be described by the following equations.

$$
\begin{aligned}
\dot{x}_{10} &= -(x_{10} - x_{11}) - f(x_{10}) \\
\dot{x}_{20} &= -(x_{20} - x_{21}) - f(x_{20}) \\
\dot{x}_{1k} &= \alpha\{(1+\gamma_C)(y_{1(k-1)} - y_{1k}) + \gamma_C(y_{2(k-1)} - y_{2k})\} \\
\dot{x}_{2k} &= \alpha\{(1+\gamma_C)(y_{2(k-1)} - y_{2k}) + \gamma_C(y_{1(k-1)} - y_{1k})\} \\
\dot{y}_{1k} &= \beta\{x_{1k} - x_{1(k+1)} - \gamma_L(x_{2k} - x_{2(k+1)})\} \\
\dot{y}_{2k} &= \beta\{x_{2k} - x_{2(k+1)} - \gamma_L(x_{1k} - x_{1(k+1)})\}
\end{aligned}
\tag{1}
$$

where $k = 1, \cdots, n$, $y_{10} = x_{10} - x_{11}$, $y_{20} = x_{20} - x_{21}$, and $x_{1(n+1)} = x_{2(n+1)} = 0$. Nonlinear function corresponding

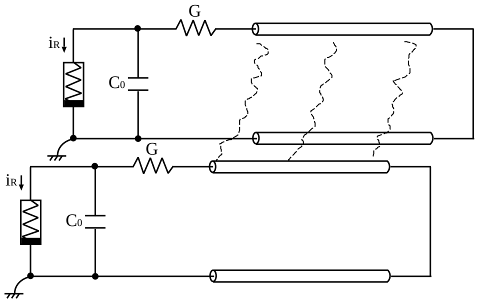


Fig. 4: Chua's circuits with transmission lines placed with same direction.

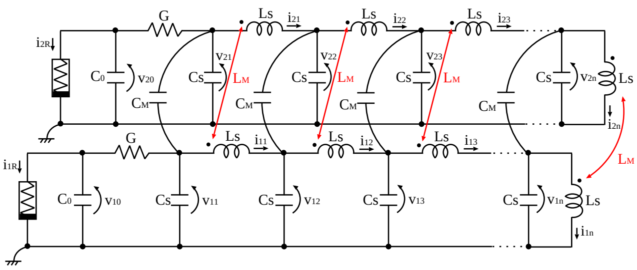


Fig. 5: Discretized circuit model of Chua's circuits with transmission lines placed with same direction.

to the $v-i$ characteristics of the nonlinear resistor can be written as

$$f(x) = bx - (a-b)\frac{|x-1| - |x+1|}{2}. \tag{2}$$

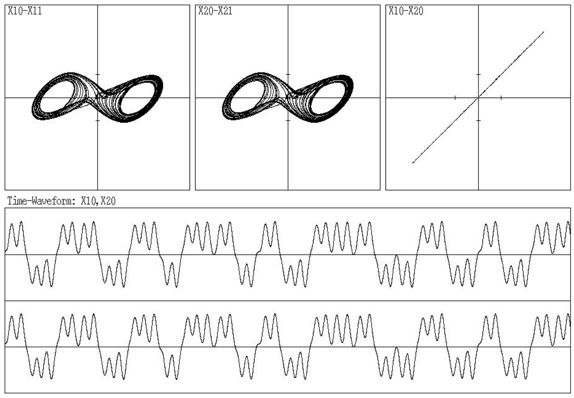
Variables and parameters are defined by the following equations.

$$\begin{aligned}
&v_{10} = Ex_{10}, \quad v_{20} = Ex_{20}, \quad v_{1k} = Ex_{1k},\\
&v_{2k} = Ex_{2k}, \quad i_{1k} = GEy_{1k}, \quad i_{2k} = GEy_{2k},\\
&a = \frac{m_0}{G}, \quad b = \frac{m_1}{G}, \quad \alpha = \frac{C_0}{C_S + 2C_M},\\
&\beta = \frac{C_0 L_S}{G^2(L_S^2 - L_M^2)}, \quad \gamma_C = \frac{C_M}{C_S}, \quad \gamma_L = \frac{L_M}{L_S},\\
&t = \frac{C_0}{G}\tau, \quad \text{“}\cdot\text{”} = \frac{d}{d\tau}, \quad (k = 1, \cdots, n).
\end{aligned} \tag{3}$$

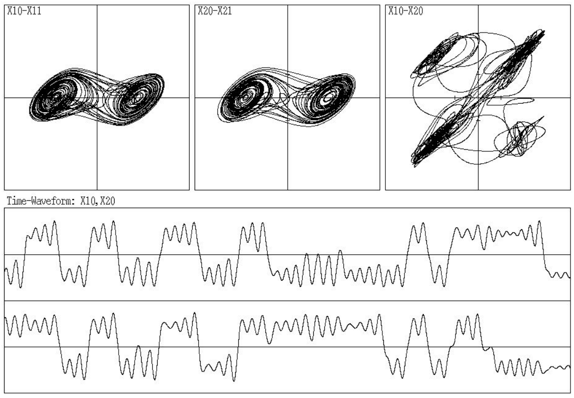
Figure 6 shows the computer calculated results of Eq. (1). We can see that the two circuits can be synchronized completely as Fig. 6(a), if the coupling is relatively large. As decreasing the coupling parameters, the synchronization becomes incomplete, although the solution tends to stay close to the synchronization plane as Fig. 6(b).

3. Chua's Circuits with Transmission Lines Placed with Opposite Direction

Next, the same two Chua's circuits with transmission lines are considered, but they are placed in parallel from the opposite direction as shown in Fig. 7.



(a) $\alpha = 0.5$, $\beta = 17.0$, $\alpha\gamma_C = \gamma_L = 0.5$.



(b) $\alpha = 0.4$, $\beta = 19.0$, $\alpha\gamma_C = \gamma_L = 0.4$.

Fig. 6: Computer simulation results. $n = 10$, $a = -1.2$, $b = -0.75$.

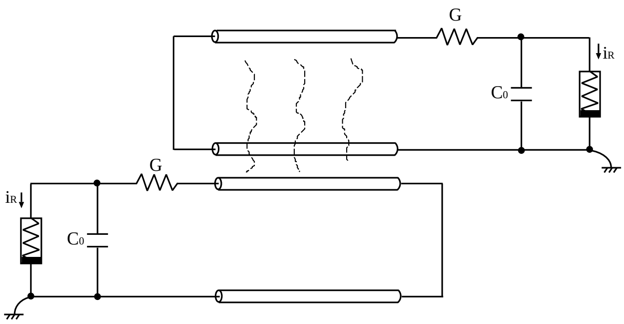


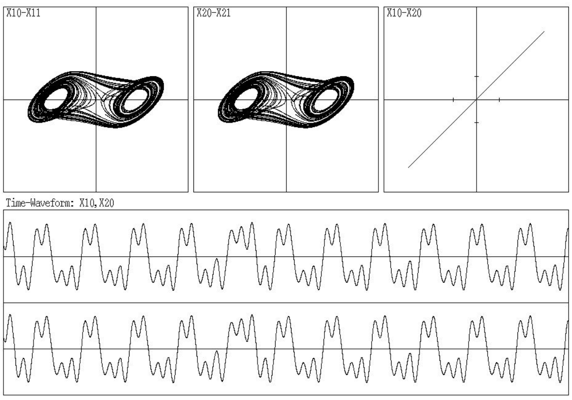
Fig. 7: Chua's circuits with transmission lines placed with opposite direction.

With the same notations of the variables and the parameters as Eq. (3), the circuit equations are derived as follows.

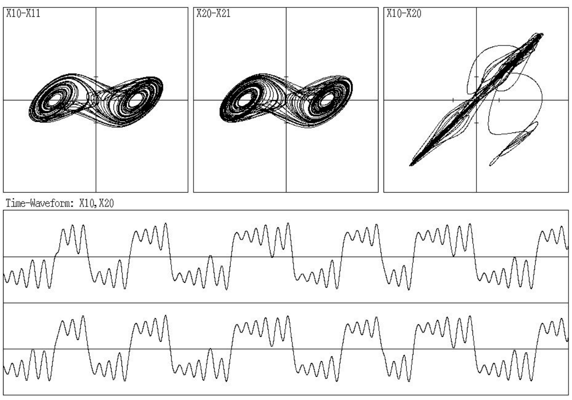
$$\begin{array}{rcl}
\dot{x}_{10} & = & -(x_{10}-x_{11})-f(x_{10}) \\
\dot{x}_{20} & = & -(x_{20}-x_{21})-f(x_{20}) \\
\dot{x}_{1k} & = & \alpha\{(1+\gamma_C)(y_{1(k-1)}-y_{1k}) \\
 & & \quad +\gamma_C(y_{2(n-k)}-y_{2(n-k+1)})\} \\
\dot{x}_{2k} & = & \alpha\{(1+\gamma_C)(y_{2(k-1)}-y_{2k}) \\
 & & \quad +\gamma_C(y_{1(n-k)}-y_{1(n-k+1)})\} \\
\dot{y}_{1j} & = & \beta\{x_{1k}-x_{1(k+1)}+\gamma_L(x_{2(n-k)}-x_{2(n-k+1)})\} \\
\dot{y}_{2j} & = & \beta\{x_{2k}-x_{2(k+1)}+\gamma_L(x_{1(n-k)}-x_{1(n-k+1)})\} \\
\dot{y}_{1n} & = & \beta(1-\gamma_L^2)x_{1n} \\
\dot{y}_{2n} & = & \beta(1-\gamma_L^2)x_{2n}
\end{array} \tag{4}$$

where $j=1,\cdots,n-1$, $k=1,\cdots,n$, $y_{10}=x_{10}-x_{11}$, and $y_{20}=x_{20}-x_{21}$.

Figure 8 shows the computer calculated results of Eq. (4). Similar to the case of the same direction, the two circuits can be synchronized, even though the transmission lines are placed from the opposite direction.



(a) $\alpha=0.5$, $\beta=17.0$, $\alpha\gamma_C=\gamma_L=0.5$.



(b) $\alpha=0.4$, $\beta=19.0$, $\alpha\gamma_C=\gamma_L=0.47$.

Fig. 8: Computer simulation results. $n=10$, $a=-1.2$, $b=-0.75$.

4. Synchronization Plane

In order to analyze the synchronization theoretically, the synchronization plane can be considered.

The following dependent variables are defined.

$$\begin{array}{rcl}
p_{xk} & = & x_{1k}-x_{2k} \\
p_{yk} & = & y_{1k}-y_{2k}
\end{array} \tag{5}$$

where $k=0,\cdots,n$. The difference system can be derived from Eq.(1) and is described using the variables as follows.

$$\begin{array}{rcl}
\dot{p}_{x0} & = & -(p_{x0}-p_{x1})-\{f(x_{10})-f(x_{20})\} \\
\dot{p}_{xk} & = & \alpha(p_{y(k-1)}-p_{yk}) \\
\dot{p}_{yk} & = & \beta(1+\gamma_L)(p_{xk}-p_{x(k+1)})
\end{array} \tag{6}$$

where $k=1,\cdots,n$ and $p_{x(n+1)}=0$. Since the term $f(x_{10})-f(x_{20})$ becomes ap_{x0} or bp_{x0} when the solution stays in the same linear region, the solution $p_{xk}=p_{yk}=0$ for all k apparently satisfies Eq. (6). This means that the solution of Eq. (1) exists on the in-phase synchronization space.

Next, let us consider the case of the opposite direction. Using the same method, the difference system of Eq.(4) can be derived as follows.

$$\begin{array}{rcl}
\dot{p}_{x0} & = & -(p_{x0}-p_{x1})-\{f(x_{10})-f(x_{20})\} \\
\dot{p}_{xk} & = & \alpha\{(1+\gamma_C)(p_{y(k-1)}-p_{yk}) \\
 & & \quad -\gamma_C(p_{y(n-k)}-p_{y(n-k+1)})\} \\
\dot{p}_{yj} & = & \beta\{p_{xj}-p_{x(j+1)}-\gamma_L(p_{x(n-j)}-p_{x(n-j+1)})\} \\
\dot{p}_{yn} & = & \beta(1+\gamma_L^2)p_{xn}
\end{array} \tag{7}$$

where $j=1,\cdots,n-1$ and $k=1,\cdots,n$. Same as the previous case, the solution $p_{xk}=p_{yk}=0$ for all k satisfy Eq. (7) and the solution of Eq. (4) also exists on the in-phase synchronization space.

5. Conclusions

In this study, two Chua's circuits with lossless transmission lines placed in parallel have been investigated. By computer simulations, it was confirmed that two Chua's circuits could be synchronized by the effect of the crosstalk. Further, the existence of the solution on the in-phase synchronization could be explained theoretically.

Investigating the stability of the synchronization plane is one of important future problems.

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