Synchronization in two van der Pol Oscillators with Noise Coupled by Time-Varying Resistor

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1. Introduction

We explore the behavior of two van der Pol oscillators coupled by a stochastically time-varying resistor. We observe switching between in-phase and anti-phase synchronization and analyze the statistics of the switching, with and without additional noise.

2. Circuit Model

We study a circuit of two (up to later added noise) identical van der Pol oscillators, coupled by a stochastically time-varying resistor (STVR), see Fig. 1. The characteristics of the STVR is shown in Fig. 2. In the time interval $[k\pi/\omega_t, (k+1)\pi/\omega_t]$, $R(t)$ is piecewise constant, taking one of the values $\{r, -r\}$. The event probabilities p_+ and p_- for tasking the values r or $-r$, respectively, satisfy the equation $p_+ + p_- = 1.0$.

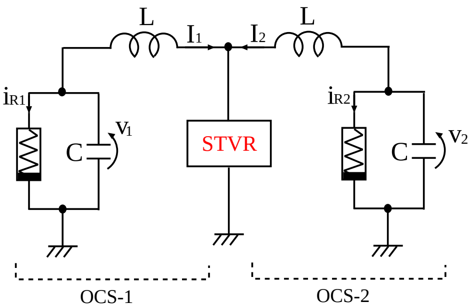

Fig. 1: Circuit model.

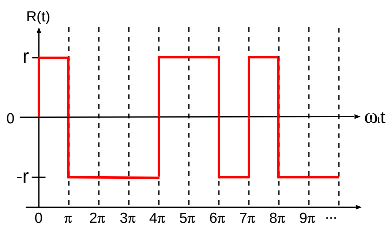

Fig. 2: Characteristics of STVR.

The normalized circuit equations obtain their form

$$
\begin{cases}
\dfrac{dx_k}{d\tau} = \varepsilon x_k (1 - {x_k}^2) - y_k \\[2ex]
\dfrac{dy_k}{d\tau} = x_k \pm \gamma(\tau) \displaystyle\sum_{j=1}^{2} y_j \qquad (k = 1, 2\ .)
\end{cases} \tag{1}
$$

In Eq. 1, ε embodies to the nonlinearity of van der Pol oscillator and $\gamma(\tau)$ is the characteristics of STVR.

3. Synchronization Phenomena

For the following computer simulations, we fix the circuit system parameters at $\varepsilon = 2.0$, $\gamma = 0.1$ and $\omega = 1.5$. Figure 3 shows the discrete data $x_1 - x_2$ between two van der Pol oscillators at $\tau = k\pi/\omega$. From this figure we can see that the switching synchronization between the in-phase and the anti-phase state is observed.

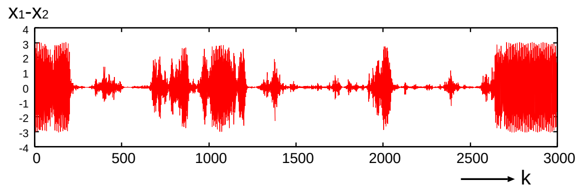

Fig. 3: Difference $x_1 - x_2$ of the two coupled oscillators.

In realistic systems, noise is omnipresent. Therefore, it is important to study the effect of noise on our findings. When extended by noise contributions to the voltage of the two capacitances. The two complementary behaviors are now investigated at the switching probability $p_+ = 0.5$, where we are also interested in the influence that a correlation between the two noise contributions would have. As a function of the percentage of shared vs. independently chosen noise contributions, we obtain the results as displayed in Fig.**??**. For independent noise contributions, we observe a stochastic resonance phenomenon for in-phase and anti-phase synchronization regimes.

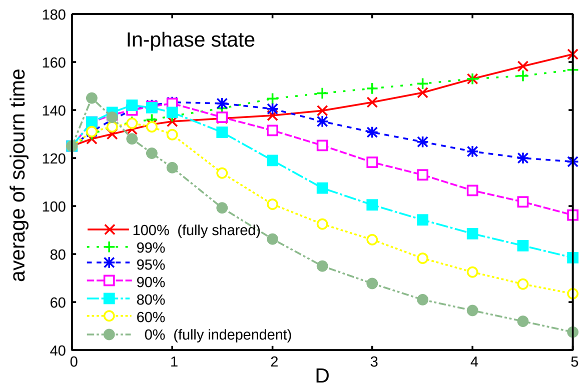

Fig. 4: Average sojourn time of in-phase by changing noise intensity.

4. Conclusions

In this study, we have investigated the switching phase state of two van der Pol oscillators coupled by STVR. The stochastic resonant behavior of synchronization was observed when the strength of the adding noise is small. We have confirmed that correlating the two noise sources quickly destroys the stochastic resonance phenomenon in the anti-phase synchronization regime.

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