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Quasi Synchronization Phenomena in Coupled Chaotic Systems as a Ladder

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***Abstract*— Some chaotic systems have two coexisting attractors depending on initial values in same parameters. In these systems, changing parameters causes switching phenomena of coexisting attractors. In this study, we investigate switching phenomena in coupled chaotic systems as a ladder. We define a quasi synchronization and confirmed the quasi synchronization phenomena in proposed systems.**

I. INTRODUCTION

There are many studies about coupled chaotic systems. Especially, many kinds of systems were proposed in a area of electrical engineering [1]-[3]. In some of these studies, many interesting phenomena which is concerned in a synchronization were investigated in detail.

On the other hand, some chaotic systems have two coexisting attractors depending on initial values in same parameters [4]-[6]. In these systems, changing parameters causes switching phenomena of coexisting attractors. We consider that matching these two attractors in coupled systems means one of quasi synchronization phenomena. By the confirmation of this phenomena, it is expected that new interesting phenomena are observed in coupled systems.

In this study, three coupled chaotic systems are investigated in order to confirm the quasi synchronization phenomena. Three systems are two coupled map lattices and one coupled chaotic circuits. Coupled map lattice [7] is a discrete time system and coupled chaotic circuits is a continuous time system. This differences is also important point in this study.

II. COUPLED CHAOTIC SYSTEMS

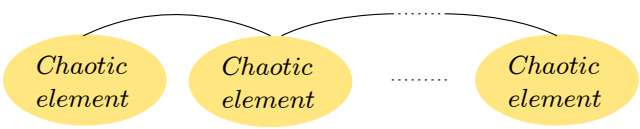

Fig. 1. System Model.

Coupled chaotic system as a ladder shown in Fig. 1 is investigated in this study. Three kinds of coupled elements are applied in the system. Each system is shown as following subsections.

A. Coupled map lattice using a cubic function

The coupled element is an one-dimensional map shown Fig. 2. The function is described as Eq. 1. Switching phenomena (red and blue) are observed in this map. The coupled system consists of this map. Namely, the system is a coupled map lattice using a cubic function.

$$f(x) = -ax^3 + ax. \tag{1}$$

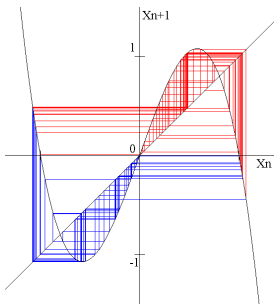

Fig. 2. The one-dimensional maps described by Eq. (1). $a = 2.8$ and $x_0 = 0.4$.

The system is written as

$$x_{n+1}(i) = (1-\varepsilon)f(x_n(i)) + \frac{\varepsilon}{2}[f(x_n(i-1)) + f(x_n(i+1))]. \tag{2}$$

where n is a discrete time step and i is a lattice point. The coupling strength is determined by parameter ε.

B. Coupled map lattice using a piece-wise linear function

The coupled element is an one-dimensional map shown as Fig. 3. A function of the map in described as Eq. 3. Switching phenomena (red and blue) are observed in this map. The coupled system is a CML using a piece-wise linear function.

$$g(x) = -ax + a|x + \frac{1}{2}| - a|x - \frac{1}{2}|. \tag{3}$$

The system is written as

$$x_{n+1}(i) = (1-\varepsilon)g(x_n(i)) + \frac{\varepsilon}{2}[g(x_n(i-1)) + g(x_n(i+1))]. \tag{4}$$

where n is a discrete time step and i is a lattice point. The coupling strength is determined by parameter ε.

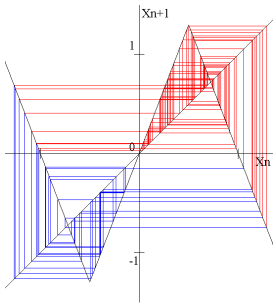

Fig. 3. The one-dimensional maps described by Eq. (3). $a = 2.8$ and $x_0 = 0.4$.

C. Coupled chaotic circuits

The coupled element is shown in Fig. 4. The circuit equation is described as follows.

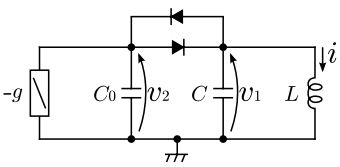

Fig. 4. Shinriki-Mori chaotic circuit.

$$\begin{cases} L\dfrac{di}{dt} &= v_1, \\ C\dfrac{dv_1}{dt} &= -i - i_d, \\ C_0\dfrac{dv_2}{dt} &= gv_2 + i_d, \end{cases} \tag{5}$$

where,

$$i_d = \begin{cases} a(v_1 - v_2 - V_{th}), & v_1 - v_2 > V_{th}, \\ 0, & -V_{th} \le v_1 - v_2 \le V_{th}, \\ a(v_1 - v_2 + V_{th}), & v_1 - v_2 < -V_{th}. \end{cases} \tag{6}$$

The diode is modeled as a piece-wise linear function. Figure 5 shows a computer simulation result. Switching phenomena (red and blue) are observed.

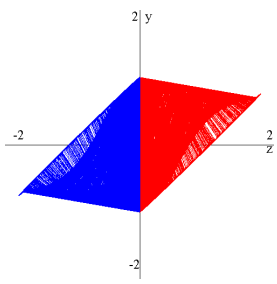

Fig. 5. Computer simulation results of the chaotic circuit. $\alpha = 10.0$, $\beta = 0.56$ and $\gamma = 2.605$.

The coupled system consists of chaotic circuits and resistors as coupling elements. The circuit model is shown in Fig. 6. By using these model, the equation of the coupled system is described as follows.

$$\begin{cases} L\dfrac{di_n}{dt} &= v_{n1}, \\ C\dfrac{dv_{n1}}{dt} &= -i_n - i_{dn} + G(v_{(n-1)1} - 2v_{n1} + v_{(n+1)1}) \\ C_0\dfrac{dv_{n2}}{dt} &= gv_{n2} + i_{dn}, \end{cases} \tag{7}$$

where,
$n = 1, 2, \cdots N$

$$i_d = \begin{cases} a(v_{n1} - v_{n2} - V_{th}), & v_{n1} - v_{n2} > V_{th}, \\ 0, & -V_{th} \le v_{n1} - v_{n2} \le V_{th}, \\ a(v_{n1} - v_{n2} + V_{th}), & v_{n1} - v_{n2} < -V_{th}. \end{cases} \tag{8}$$

By changing parameters and variables,

$$\begin{aligned} &x_n = \frac{1}{V_{th}}\sqrt{\frac{L}{C}} \cdot i, \quad y_n = \frac{1}{V_{th}} \cdot v_{n1}, \quad z_n = \frac{1}{V_{th}} \cdot v_{n2} \\ &\alpha = a\sqrt{\frac{L}{C}}, \quad \beta = g\frac{C}{C_0}\sqrt{\frac{L}{C}}, \quad \gamma = \frac{C}{C_0}, \quad \delta = G\sqrt{\frac{L}{C}}, \\ &\tau = \frac{1}{\sqrt{LC}}t, \quad \frac{d}{d\tau} = \text{" . "} \end{aligned} \tag{9}$$

Normalized circuit equations are described as follows.

$$\begin{cases} \dot{x}_n &= y_n, \\ \dot{y}_n &= -x_n - \alpha f(y_n - z_n) + \delta(y_{n-1} - 2y_n + y_{n+1}), \\ \dot{z}_n &= \beta z_n + \alpha\gamma f(y_n - z_n), \end{cases} \tag{10}$$

where,
$n = 1, 2, \cdots N$

$$f(y_n - z_n) = \begin{cases} y_n - z_n - 1, & y_n - z_n > 1, \\ 0, & -1 \le y_n - z_n \le 1, \\ y_n - z_n + 1, & y_n - z_n < -1. \end{cases} \tag{11}$$

III. COMPUTER SIMULATIONS

We carried out computer simulations of three models. Figure 8 shows one of the computer simulation results in the case of a coupled map lattice using a cubic function. Time series of each coupled element are shown in Fig. 8 (a). Red and blue lines show differences of two attractors. Differences between two elements are shown in Fig. 8 (b). Green lines show differences of two attractors. Switching phenomena are observed.

Figure 9 shows one of the computers simulation result in the case of coupled map lattice using a piece-wise linear function. Time series and differences of each element are shown in Fig. 9 (a) and (b), respectively. Switching phenomena are

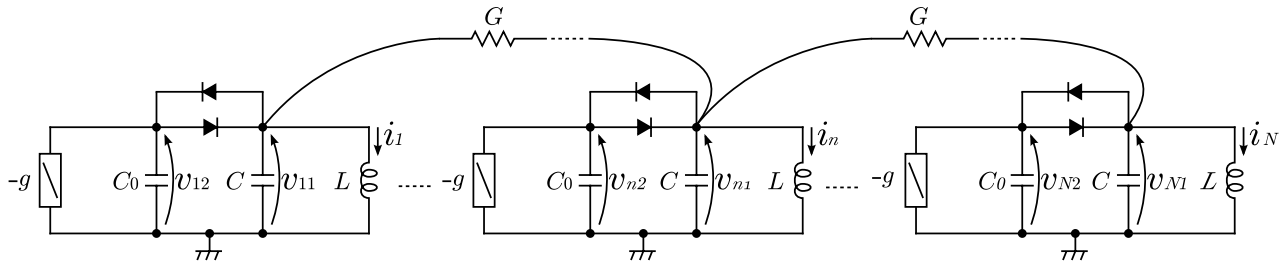

Fig. 6. Circuit Model.

also observed. Additionally, synchronization phenomena are observed. When switching state of two elements is the same, synchronization states (purple circle) and asynchronization states (light blue circle) are confirmed like as shown in Fig. 7. We define a quasi synchronization as the state that the switching state is the same and two elements do not be synchronized.

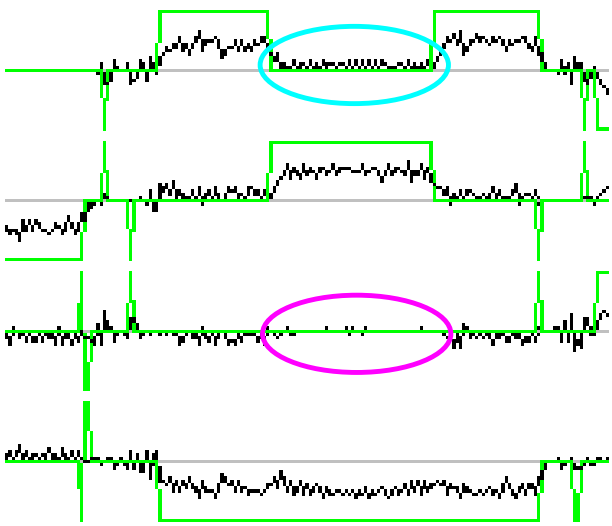

Fig. 7. Magnification of Fig.9 (b). A quasi synchronization is shown in a light blue circle. A synchronization is shown in a purple circle.

Figures 10 show one of the computers simulation result in the case of coupled chaotic circuits. Time series and Differences are shown in Fig. 10 (a) and (b), respectively. Vertical axes is z_n and horizontal axes is time. Quasi synchronizing phenomena are also observed in this case.

We have ever observed this quasi synchronization in the case of coupled map lattice using a cubic function. In this case, synchronization phenomena can be observed by changing parameters. However, quasi synchronization phenomena can be observed. Now, we continue seeking the phenomena. If we can obtain the phenomena, we will show the result in our presentation.

We could observed the quasi synchronization phenomena in two cases. These two cases are discrete and continuous time systems. It means that this phenomena can be observed many kinds of coupled chaotic systems.

IV. CONCLUSIONS

The quasi synchronization phenomena observed in coupled chaotic systems were investigated. Three kind of systems were investigated and quasi synchronization phenomena were confirmed in two systems. By confirmation of phenomena, we can investigate coupled systems from a new point of view.

Our future works are investigating the relationship between parameters and quasi synchronization phenomena and investigating other coupled systems.

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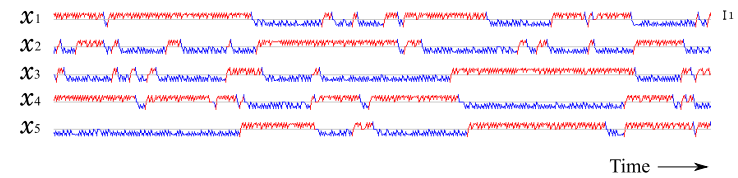
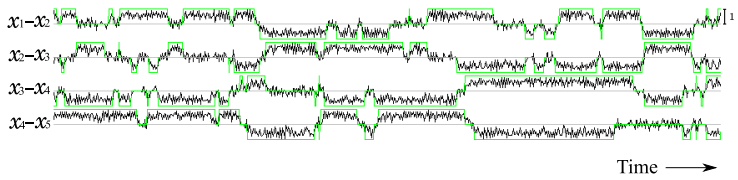
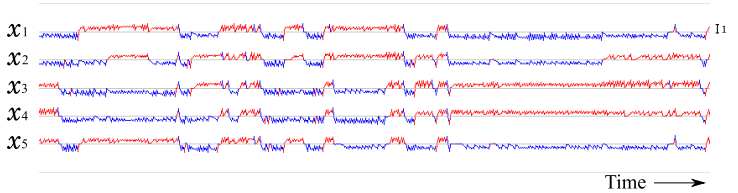


(a) Time series of each coupled elements.

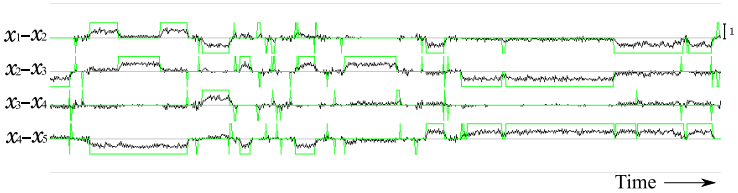


(b) Differences between two elements.

Fig. 8. Computer simulation result of coupled map lattice using a cubic function. $a = 2.80$ and $\varepsilon = 0.07$.

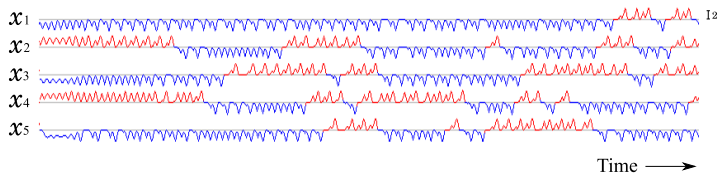


(a) Time series of each coupled elements.

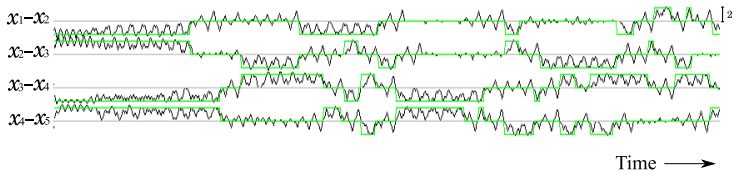


(b) Differences between two elements.

Fig. 9. Computer simulation result of coupled map lattice using a piece-wise linear function. $a = 2.48$ and $\varepsilon = 0.40$.



(a) Time series of each coupled elements.



(b) Differences between two elements.

Fig. 10. Computer simulation result of coupled chaotic circuits. $\alpha = 10.0$, $\beta = 0.55$, $\gamma = 2.605$ and $\delta = 0.1$.