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Relationship Between Oscillatory Phenomena and Boundary Conditions in Two-template CNN

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***Abstract*—In our previous study, we proposed a two-template CNN. We investigated oscillatory phenomena in two-template CNN. As a result, it was confirmed that the boundary condition of CNN affects the oscillatory phenomena. In this study, we investigate relationship between oscillatory phenomena and boundary conditions. We consider that investigating the relationship between oscillatory phenomena and boundary conditions are important to understanding coupled oscillatory systems. Because we consider that this system is a new class of coupled oscillatory systems.**

I. INTRODUCTION

Cellular Neural Networks (CNNs) [1]-[3] are one kind of mutually coupled neural networks. The main characteristics are the local connection and the parallel signal processing. There have been many studies on CNNs and many kinds of CNNs have been proposed. One of them is two-layer CNN [4]. Two-layer CNN can generate many interesting phenomena. For instance, self-organizing pattern, active wave propagation [5] and so on are observed. Like this, some kinds of CNNs generate interesting phenomena.

In our previous study [6], A CNN using two kinds of templates (two-template CNN) was proposed. Cells having one template and the other are placed as checkered. The system generates similar phenomena to Two-layer CNN. For instance, self-organizing patterns, active wave propagation phenomena, clustering phenomena and oscillatory phenomena were observed. Because the structure is simpler than Two-Layer CNN, the implementation is easier than Two-Layer CNN.

We consider that this system is a new class of coupled oscillatory system. Because oscillatory factors are coupled checkered by using two templates. Thus, we consider that investigating oscillatory phenomena is very important work. Additionally, realizing this structure by using oscillatory circuit is not so easy. Before now, a relationship between oscillatory phenomena and initial values was investigated in detail. As a result, we confirmed that the boundary condition of CNN affects the oscillatory phenomena.

In this study, we investigate the relationship between oscillatory phenomena and boundary conditions. In the case of normal CNN, oscillatory phenomena could not generated by the case of symmetry and linear templates. From this point of view, boundary conditions play a role of asymmetrization of the system are guessed.

II. CNN USING TWO KINDS OF TEMPLATES

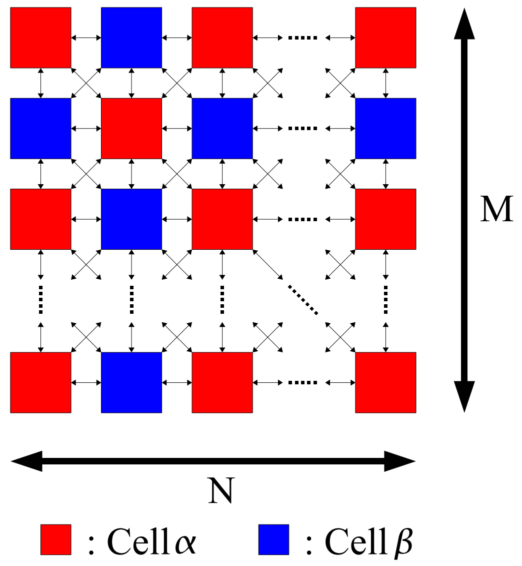


Fig. 1. System model of CNN using two kinds of templates.

Figure 1 shows a system model of proposed system. We assume that the system has a two-dimensional M by N array structure. Each cell in the array is denoted as $c(i,j)$, where (i,j) is the position of the cell, $1 \leq i \leq M$ and $1 \leq j \leq N$. The coupling radius is assumed to be one. In this proposed CNN, two kinds of templates are used. Cells having one template are called as Cell α and the other are called as Cell β. These two types of the cells are placed as checkered. The state equations of the cells are given as follows:

1: The case that $i+j$ is an even number.

$$\frac{dx_{ij}}{dt} = -x_{ij} + I_\alpha + \sum_{c(k,l)} A_\alpha(i,j;k,l)y_{kl} + \sum_{c(k,l)} B_\alpha(i,j;k,l)u_{kl} \quad (1)$$

2: The case that $i+j$ is an odd number.

$$\frac{dx_{ij}}{dt} = -x_{ij} + I_\beta + \sum_{c(k,l)} A_\beta(i,j;k,l)y_{kl} + \sum_{c(k,l)} B_\beta(i,j;k,l)u_{kl} \quad (2)$$

$A_{\{\alpha\beta\}}(i,j;k,l)y_{kl}$, $B_{\{\alpha\beta\}}(i,j;k,l)u_{kl}$ and $I_{\{\alpha\beta\}}$ are called as the feedback coefficient, the control coefficient and the bias current, respectively. The output equation of the cell is given as follows:

$$y_{ij} = f(x_{ij}). \quad (3)$$

where,

$$f(x) = 0.5(|x+1| - |x-1|). \quad (4)$$

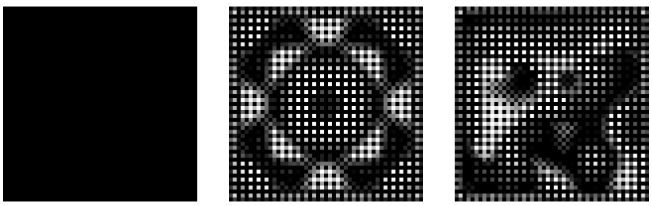
The variables u and y are the input and output variables of the cell, respectively. $\boldsymbol{A}_\alpha$, $\boldsymbol{B}_\alpha$, $\boldsymbol{A}_\beta$ and $\boldsymbol{B}_\beta$ are 3 times 3 matrices, which can be described to have a similar form to Eq. (5).

$$\begin{pmatrix} A_\alpha(i,j;i-1,j-1) & A_\alpha(i,j;i-1,j) & A_\alpha(i,j;i-1,j+1) \\ A_\alpha(i,j;i,j-1) & A_\alpha(i,j;i,j) & A_\alpha(i,j;i,j+1) \\ A_\alpha(i,j;i+1,j-1) & A_\alpha(i,j;i+1,j) & A_\alpha(i,j;i+1,j+1) \end{pmatrix} \quad (5)$$

This proposed system is more complex than the normal CNNs. This system has a unique characteristic. Namely, a pair of cell α and cell β are needed for a simple oscillation. Additionally, one cell α connects with four neighbor cells β and one cell β also connects with four neighbor cells α. Like this, these cells are sharing a factor of oscillation. This type of connection may be difficult to realize by coupling normal oscillators. Hence, we consider that this system is a new class of coupled oscillatory systems.

III. RELATIONSHIP BETWEEN OSCILLATORY PHENOMENA AND BOUNDARY CONDITION

In our previous study, we observed oscillatory phenomena shown in Fig. 2. Figure 2 (a) shows initial state which is set as 1. Figure 2 (b) and (c) show snapshots. We can observe asymmetry oscillation phenomena like as Fig. 2 (b) via symmetry oscillation like as Fig. 2 (c).



(a) Initial state (b) 10^4 iteration (c) 10^5 iteration

Fig. 2. Oscillatory phenomena for the CNN array consists of 49 × 49. $p = 1$ and $q = 1$.

In this study, we investigate about five kinds of fixed boundary conditions and a periodic boundary condition. The initial state values set as 1. The CNN array consists of 15 × 15. The template is set as

$$\boldsymbol{A}_\alpha = \begin{pmatrix} -p & p & -p \\ p & p & p \\ -p & p & -p \end{pmatrix}, \quad \boldsymbol{A}_\beta = \begin{pmatrix} p & -p & p \\ -p & -p & -p \\ p & -p & p \end{pmatrix}, \quad (6)$$
$$\boldsymbol{B}_\alpha = 0, \quad \boldsymbol{B}_\beta = 0, \quad I_\alpha = q, \quad I_\beta = q,$$

where p and q are corresponding to the coupling factors and biases I_α and I_β, respectively. Template parameter p set as 0.5, 1, 1.5, ..., 6, and bias q set as 0, 0.5, 1, ..., 13. By using these conditions and parameters, computer simulations are carried out. Some of these conditions were carried out in our previous study. As a result of previous study, it is clear that two kind of oscillatory phenomena are observed. One is the case that phenomena are observed in all cells. Another is the case that phenomena are observed in around boundary cells only.

Figures 3 – 8 show simulation results. Horizontal axis is the value of p. Vertical axis is the value of q. "A" means that the oscillatory phenomena are observed in all cells. "R" means that the oscillatory phenomena are observed in boundary cells only. "N" means that the oscillatory phenomena are not observed. Figure 3 shows the case that boundary conditions are set as 0. Two kinds of oscillatory phenomena are observed and the area is divided two areas. One is observed in $1.5 \leq p \leq 6$ and $0 \leq q \leq 0.5$. In this area, oscillatory phenomena "R" are observed. Another is observed in the area which satisfies a fixed ratio of p and q. In this area, oscillatory phenomena "A" are observed. Figure 4 shows the case that boundary conditions are set as 1. Two kinds of oscillatory phenomena are observed. The area of "R" is larger than the case of Fig. 3. Figure 5 shows the case that boundary conditions are set as -1. Two regions of oscillatory phenomena are observed. However, the region of "R" is very smaller than the cases of Fig. 3 and Fig. 4. Figure 6 shows the case that boundary conditions of virtual cells α are set as 1 and boundary conditions of virtual cells β are set as -1. In this case, phenomena "R" are observed in large area compared with Figs. 3 – 5. Figure 7 shows the case that boundary conditions of virtual cells α are set as -1 and boundary conditions of virtual cells β are set as 1. In this case, phenomena "R" are observed in $p = 1.5$ and $q = 0$ only. Figure 8 shows the case of the periodic boundary condition. Namely, the shape of CNN becomes torus and there are no virtual cells. In this case, the CNN array consists of 14 × 14 because of keeping symmetrical property of the system. In this case, oscillatory phenomena are not observed.

From these results, it is considered that boundary conditions affects oscillatory phenomena. Especially, conditions of virtual cells α are very strong. Additionally, it is considered that boundary cells are needed for oscillatory phenomena. Because oscillatory phenomena are not observed in the case of the periodic boundary condition.

IV. CONCLUSIONS

In this study, the relationship between oscillatory phenomena and boundary conditions of two-template CNN was investigated. Computer simulations were carried out about six kinds of boundary conditions.

As a result, we confirmed that oscillatory phenomena of the system strongly depends on these boundary conditions. Especially, conditions of virtual cells α are very strong and boundary cells are needed for oscillatory phenomena. However, we did not investigate the case of the Zero-flux boundary conditions and the other initial conditions. Additionally, we limited values of templates. These conditions should be investigated.

In our future works, we will investigate these conditions and compare this system with other coupled systems.

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13	N	N	N	N	N	N	N	N	N	N	N	N
12.5	N	N	N	N	N	N	N	N	N	N	N	N
12	N	N	N	N	N	N	N	N	N	N	N	A
11.5	N	N	N	N	N	N	N	N	N	N	N	N
11	N	N	N	N	N	N	N	N	N	N	A	N
10.5	N	N	N	N	N	N	N	N	N	N	N	N
10	N	N	N	N	N	N	N	N	N	A	N	N
9.5	N	N	N	N	N	N	N	N	N	N	N	N
9	N	N	N	N	N	N	N	N	A	N	N	N
8.5	N	N	N	N	N	N	N	N	N	N	N	N
8	N	N	N	N	N	N	N	A	N	N	N	N
7.5	N	N	N	N	N	N	N	N	N	N	N	N
7	N	N	N	N	N	N	A	N	N	N	N	N
6.5	N	N	N	N	N	N	N	N	N	N	N	N
6	N	N	N	N	N	A	N	N	N	N	N	N
5.5	N	N	N	N	N	N	N	N	N	N	N	N
5	N	N	N	N	A	N	N	N	N	N	N	N
4.5	N	N	N	N	N	N	N	N	N	N	N	N
4	N	N	N	A	N	N	N	N	N	N	N	N
3.5	N	N	R	N	N	N	N	N	N	N	N	N
3	N	N	A	N	N	N	N	N	N	N	N	N
2.5	N	A	A	N	N	N	N	N	N	N	N	N
2	N	A	N	N	N	N	N	N	N	N	N	N
1.5	A	A	A	N	N	N	N	N	N	N	N	N
1	A	A	N	N	N	N	N	N	N	N	N	N
0.5	A	N	R	R	R	R	R	R	R	R	R	R
0	A	N	R	R	R	R	R	R	R	R	R	R
	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6

Fig. 3. Relationship between oscillatory phenomena and parameters. Horizontal axis is p and vertical axis is q. The boundary condition is set as 0.

13	N	N	N	N	N	N	N	N	N	N	N	N
12.5	N	N	N	N	N	N	N	N	N	N	N	N
12	N	N	N	N	N	N	N	N	N	N	N	N
11.5	N	N	N	N	N	N	N	N	N	N	N	N
11	N	N	N	N	N	N	N	N	N	N	N	A
10.5	N	N	N	N	N	N	N	N	N	N	N	N
10	N	N	N	N	N	N	N	N	N	N	A	N
9.5	N	N	N	N	N	N	N	N	N	A	N	N
9	N	N	N	N	N	N	N	N	N	N	N	N
8.5	N	N	N	N	N	N	N	N	A	N	N	N
8	N	N	N	N	N	N	N	N	N	N	N	N
7.5	N	N	N	N	N	N	N	A	N	N	N	N
7	N	N	N	N	N	N	N	N	N	N	N	N
6.5	N	N	N	N	N	N	A	N	N	N	N	N
6	N	N	N	N	N	N	N	N	N	N	N	N
5.5	N	N	N	N	N	A	N	N	N	N	N	N
5	N	N	N	N	A	N	N	N	N	N	N	N
4.5	N	N	N	N	A	N	N	N	N	N	N	N
4	N	N	N	A	N	N	N	N	N	N	N	N
3.5	N	N	N	A	N	N	N	N	N	N	N	N
3	N	N	A	N	N	N	N	N	N	N	N	N
2.5	N	A	A	N	N	N	N	N	N	N	N	N
2	N	A	N	N	N	N	N	N	N	N	N	R
1.5	A	A	N	N	N	R	R	R	R	R	R	R
1	A	A	R	N	R	R	R	R	R	R	R	R
0.5	A	N	R	R	R	R	R	R	R	R	R	R
0	A	R	R	R	R	R	R	R	R	R	R	R
	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6

Fig. 4. Relationship between oscillatory phenomena and parameters. Horizontal axis is p and vertical axis is q. The boundary condition is set as 1.

	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
13	N	N	N	N	N	N	N	N	N	N	N	A
12.5	N	N	N	N	N	N	N	N	N	N	N	N
12	N	N	N	N	N	N	N	N	N	N	A	N
11.5	N	N	N	N	N	N	N	N	N	N	N	N
11	N	N	N	N	N	N	N	N	N	N	N	N
10.5	N	N	N	N	N	N	N	N	N	A	N	N
10	N	N	N	N	N	N	N	N	N	N	N	N
9.5	N	N	N	N	N	N	N	N	A	N	N	N
9	N	N	N	N	N	N	N	N	N	N	N	N
8.5	N	N	N	N	N	N	N	A	N	N	N	N
8	N	N	N	N	N	N	N	N	N	N	N	N
7.5	N	N	N	N	N	N	A	N	N	N	N	N
7	N	N	N	N	N	N	N	N	N	N	N	N
6.5	N	N	N	N	N	A	N	N	N	N	N	N
6	N	N	N	N	N	N	N	N	N	N	N	N
5.5	N	N	N	N	R	N	N	N	N	N	N	N
5	N	N	N	N	A	N	N	N	N	N	N	N
4.5	N	N	N	R	N	N	N	N	N	N	N	N
4	N	N	N	A	N	N	N	N	N	N	N	N
3.5	N	N	R	N	N	N	N	N	N	N	N	N
3	N	N	A	N	N	N	N	N	N	N	N	N
2.5	N	A	N	N	N	N	N	N	N	N	N	N
2	N	A	N	N	N	N	N	N	N	N	N	N
1.5	A	A	A	N	N	N	N	N	N	N	N	N
1	A	A	N	N	N	N	N	N	N	N	N	N
0.5	A	N	R	R	R	R	N	N	N	N	N	N
0	A	N	R	N	R	N	N	N	N	N	N	N

Fig. 5. Relationship between oscillatory phenomena and parameters. Horizontal axis is p and vertical axis is q. The boundary condition is set as -1.

	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
13	N	N	N	N	N	N	N	N	N	N	N	N
12.5	N	N	N	N	N	N	N	N	N	N	N	N
12	N	N	N	N	N	N	N	N	N	A	N	N
11.5	N	N	N	N	N	N	N	N	N	N	N	N
11	N	N	N	N	N	N	N	N	A	N	N	N
10.5	N	N	N	N	N	N	N	N	N	N	N	N
10	N	N	N	N	N	N	N	N	N	N	N	N
9.5	N	N	N	N	N	N	N	A	N	N	N	N
9	N	N	N	N	N	N	N	N	N	N	N	N
8.5	N	N	N	N	N	N	R	N	N	N	N	N
8	N	N	N	N	N	N	N	N	N	N	N	N
7.5	N	N	N	N	N	R	N	N	N	N	N	N
7	N	N	N	N	N	A	N	N	N	N	N	N
6.5	N	N	N	N	N	N	N	N	N	N	N	N
6	N	N	N	N	R	N	N	N	N	N	N	R
5.5	N	N	N	N	N	N	N	N	N	N	R	R
5	N	N	N	R	N	N	N	N	N	R	R	R
4.5	N	N	N	R	N	N	N	N	R	R	R	R
4	N	N	R	N	N	N	N	R	R	R	R	R
3.5	N	N	R	N	N	R	R	R	R	R	R	R
3	N	N	A	N	R	R	R	R	R	R	R	N
2.5	N	R	N	R	R	R	R	R	R	N	N	N
2	N	A	R	R	R	R	R	N	N	N	R	R
1.5	A	A	N	R	R	N	N	N	R	R	R	R
1	A	A	N	R	N	N	R	R	R	R	R	R
0.5	A	R	N	R	R	R	R	R	R	R	R	R
0	A	R	R	R	R	R	R	R	R	R	R	R

Fig. 6. Relationship between oscillatory phenomena and parameters. Horizontal axis is p and vertical axis is q. Virtual cells α are set as 1 and virtual cells β are set as -1.

	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
13	N	N	N	N	N	N	N	N	N	N	N	N
12.5	N	N	N	N	N	N	N	N	N	N	N	N
12	N	N	N	N	N	N	N	N	N	N	N	N
11.5	N	N	N	N	N	N	N	N	N	N	N	N
11	N	N	N	N	N	N	N	N	N	N	N	N
10.5	N	N	N	N	N	N	N	N	N	N	N	N
10	N	N	N	N	N	N	N	N	N	N	N	N
9.5	N	N	N	N	N	N	N	N	N	N	N	A
9	N	N	N	N	N	N	N	N	N	N	N	N
8.5	N	N	N	N	N	N	N	N	N	N	A	N
8	N	N	N	N	N	N	N	N	N	A	N	N
7.5	N	N	N	N	N	N	N	N	N	N	N	N
7	N	N	N	N	N	N	N	N	A	N	N	N
6.5	N	N	N	N	N	N	N	A	N	N	N	N
6	N	N	N	N	N	N	N	N	N	N	N	N
5.5	N	N	N	N	N	N	A	N	N	N	N	N
5	N	N	N	N	N	A	N	N	N	N	N	N
4.5	N	N	N	N	N	A	N	N	N	N	N	N
4	N	N	N	N	A	N	N	N	N	N	N	N
3.5	N	N	N	A	N	N	N	N	N	N	N	N
3	N	N	A	A	N	N	N	N	N	N	N	N
2.5	N	N	A	N	N	N	N	N	N	N	N	N
2	N	A	A	N	N	N	N	N	N	N	N	N
1.5	N	A	N	N	N	N	N	N	N	N	N	N
1	A	A	N	N	N	N	N	N	N	N	N	N
0.5	A	N	N	N	N	N	N	N	N	N	N	N
0	A	N	R	N	N	N	N	N	N	N	N	N

Fig. 7. Relationship between oscillatory phenomena and parameters. Horizontal axis is p and vertical axis is q. Virtual cells α are set as -1 and virtual cells β are set as 1.

	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
13	N	N	N	N	N	N	N	N	N	N	N	N
12.5	N	N	N	N	N	N	N	N	N	N	N	N
12	N	N	N	N	N	N	N	N	N	N	N	N
11.5	N	N	N	N	N	N	N	N	N	N	N	N
11	N	N	N	N	N	N	N	N	N	N	N	N
10.5	N	N	N	N	N	N	N	N	N	N	N	N
10	N	N	N	N	N	N	N	N	N	N	N	N
9.5	N	N	N	N	N	N	N	N	N	N	N	N
9	N	N	N	N	N	N	N	N	N	N	N	N
8.5	N	N	N	N	N	N	N	N	N	N	N	N
8	N	N	N	N	N	N	N	N	N	N	N	N
7.5	N	N	N	N	N	N	N	N	N	N	N	N
7	N	N	N	N	N	N	N	N	N	N	N	N
6.5	N	N	N	N	N	N	N	N	N	N	N	N
6	N	N	N	N	N	N	N	N	N	N	N	N
5.5	N	N	N	N	N	N	N	N	N	N	N	N
5	N	N	N	N	N	N	N	N	N	N	N	N
4.5	N	N	N	N	N	N	N	N	N	N	N	N
4	N	N	N	N	N	N	N	N	N	N	N	N
3.5	N	N	N	N	N	N	N	N	N	N	N	N
3	N	N	N	N	N	N	N	N	N	N	N	N
2.5	N	N	N	N	N	N	N	N	N	N	N	N
2	N	N	N	N	N	N	N	N	N	N	N	N
1.5	N	N	N	N	N	N	N	N	N	N	N	N
1	N	N	N	N	N	N	N	N	N	N	N	N
0.5	N	N	N	N	N	N	N	N	N	N	N	N
0	N	N	N	N	N	N	N	N	N	N	N	N

Fig. 8. Relationship between oscillatory phenomena and parameters. Horizontal axis is p and vertical axis is q. The boundary condition is set as a periodic boundary condition. CNN array is set as 14 $\times$ 14.