2008 IEEE Workshop on
Nonlinear Circuit Networks, Tokushima, Dec 5-6, 2008

Suboptimal Receiver using Chaotic Sequences with Biased Values

Shintaro ARAI†, Yoshifumi NISHIO† and Takaya YAMAZATO‡
†Department of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima, 770-8506 JAPAN
{arai, nishio}@ee.tokushima-u.ac.jp
‡EcoTopia Science Institute, Nagoya University
Furo-cho, Chikusa-ku, Nagoya, 464-8603, JAPAN,
yamazato@nuee.nagoya-u.ac.jp

***Abstract*— We investigate and evaluate a suboptimal receiver using chaotic sequences with biased values. In our previous research, we investigated a differential chaos shift keying (DCSK) using chaotic sequences with biased values purposely and confirmed its better performance. However, our previous study only performed the computer simulation of DCSK using these chaotic sequences. In this study, we focus on the suboptimal receiver and evaluate its performance with chaotic sequences with biased values.**

I. Introduction

Research on digital communications systems using chaos becomes a hot topic [1]– [5]. Especially, it is attracted to develop noncoherent detection systems which do not need to recover basis signals (unmodulated carriers) at the receiver. DCSK [1] and an optimal receiver [2] are well-known as typical noncoherent systems.

Analyzing chaotic sequence as well as its behavior is essential for improving the the performance of chaos communications. A Chaotic sequence is a series of non-periodic signals generated from nonlinear dynamical systems. These signals are sensitive to initial conditions and difficult to predict the behavior of the future from the past observational signal. Also the chaotic sequence can be generated from a simple model, such as a one-dimensional chaotic map. These characteristics are advantages of using the chaotic sequence for communication systems. Thus, we consider that the utilization of characteristics of the chaotic sequence is important to improve the performance of chaos communication systems. In our previous research, we investigated the performance of DCSK using chaotic sequences with biased values purposely [7]. As results, it could be observed that its performance was better than that of the conventional method. However, our previous study only performed the simulation of DCSK using these sequences. Namely, we need to investigate chaotic sequences with biased values in other chaos communication systems.

In this study, we focus on a suboptimal receiver as one of chaos communication systems. The suboptimal receiver achieves the performance equivalent or similar to the optimal receiver using different algorithms. We apply the chaotic sequence with biased values to the suboptimal receiver and evaluate its performance.

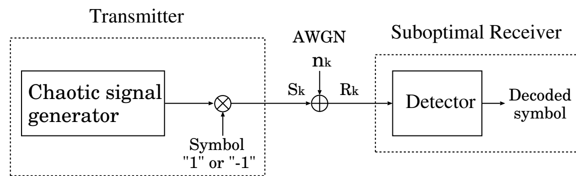


Fig. 1. Block diagram of discrete-time binary CSK communication system.

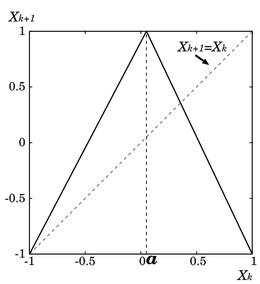


Fig. 2. Skew tent map.

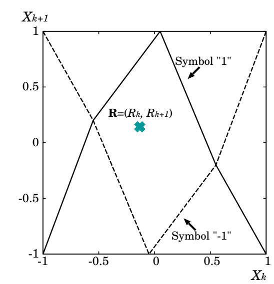


Fig. 3. Chaotic maps with some slopes.

II. System Overview

We consider a discrete-time binary CSK communication system, as shown in Fig. 1. This system consists of a transmitter, a channel and a suboptimal receiver.

In the transmitter, a chaotic sequence is generated by a chaotic map. To generate the chaotic sequence with biased values, we give some slopes to the skew tent map (Fig 2) which is well-known as a typical one-dimensional map and change slopes of map as shown in Fig 3. Figure 3 is described by

$$x_{k+1} = \begin{cases} \frac{(r_1+1)x_k - q_1 + r_1}{q_1+1} & (-1 \le x_k \le q_1) \\ \frac{(1-r_1)x_k - q_1 + ar_1}{a-q_1} & (q_1 < x_k \le a) \\ \frac{(r_2-1)x_k + q_2 - ar_2}{q_2 - a} & (a < x_k \le q_2) \\ \frac{-(r_2+1)x_k + q_2 + r_2}{1-q_2} & (q_2 < x_k \le 1) \end{cases}, \quad (1)$$

where a denotes a position of the top of the skew tent map, q_1, r_1, q_2 and r_2 are the parameters deciding the slopes. An analysis of this map is taken up in Sec. III. The transmitter generate different chaotic sequences for every symbol by changing the

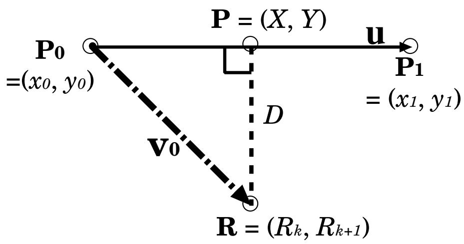


Fig. 4. Calculation of the shortest distance

initial value. The information symbol is modulated by Chaos Shift Keying (CSK) which is one of digital modulation system using chaos. If the information symbol "1" is sent, the solid map of Fig 3 is used, and if "-1" is sent, the dashed map of Fig 3 is used. To transmit a 1-bit information, N chaotic signal samples are generated, where N is chaotic sequence length. Therefore the transmitted signal is denoted by a vector $\mathbf{S} = (S_1 \ S_2 \ \cdots \ S_N)$.

In the channel, we assume the additive white Gaussian noise (AWGN) channel with a mean of zero and variance of $N_0 = \sigma^2$. Here, the noise signal is denoted by the noise vector $\mathbf{n} = (n_1 \ n_2 \ \cdots \ n_N)$. Thus, the received signal block is given by $\mathbf{R} = (R_1 \ R_2 \ \cdots \ R_N) = \mathbf{S} + \mathbf{n}$.

The suboptimal receiver achieves the performance equivalent or similar to the optimal receiver using different algorithms. The receiver has the chaotic map used for the modulation at the transmitter memorized. In this study, we use the suboptimal receiver proposed by the authors [6]. Our suboptimal receiver calculate shortest distances between the received signal and the chaotic maps and performs detection of the symbol.

Here, we introduce the operation of our suboptimal receiver. The receiver calculates the shortest distance between received signal and the maps in the N_d-dimensional space using N_d successive received signal samples (N_d : $2, 3, \cdots$). As an example, we explain the case of $N_d = 2$. In this case, we consider two successive received signal samples $\mathbf{R} = (R_k, R_{k+1})$ as coordinate of chaotic map (Fig 3) , where $k = 1, 2, \cdots, N-1$. To decide which map is closer to the point $\mathbf{R}$ in Fig. 3, the shortest distance between the point and the map has to be calculated. Therefore, the receiver can calculate the shortest distance using the scalar product of the vector.

Any two points $\mathbf{P_0} = (x_0, y_0)$ and $\mathbf{P_1} = (x_1, y_1)$ are chosen from each straight line in the space of Fig. 3, as shown in Fig. 4. Using Fig. 4, we can calculate the point $\mathbf{P} = (X, Y)$ closest to $\mathbf{R}$ and the shortest distance D using the following equations.

$$\mathbf{P} = (X, Y) = (\mathbf{u} \cdot \mathbf{v_0})\,\mathbf{u} + \mathbf{P_0} \tag{2}$$

$$\begin{aligned} D &= ||\mathbf{P} - \mathbf{R}|| \\ &= \sqrt{(X - R_k)^2 + (Y - R_{k+1})^2} \end{aligned} \tag{3}$$

where

$$\begin{aligned} \text{unit vector } \mathbf{u} &= \frac{\mathbf{P_1} - \mathbf{P_0}}{||\mathbf{P_1} - \mathbf{P_0}||} & (4) \\ \mathbf{v_0} &= \mathbf{R} - \mathbf{P_0} \ . & (5) \end{aligned}$$

Note that if the point is outside of the cube, we calculate the distance between the point and the nearest edges of the maps.

In the case of using Fig 3, there are eight straight lines in the space. Therefore, the minimum value from four distances is chosen as the shortest distance D_1 for symbol "1". In the same way, D of symbol "-1" is chosen as D_{-1}. the receiver calculates both D_1 and D_{-1} for all k and finds their summations $\sum D_1$ and $\sum D_{-1}$. Finally, we decide the decoded symbol as 1 (or -1) for $\sum D_1 < \sum D_{-1}$ (or $\sum D_1 > \sum D_{-1}$).

The calculation of the shortest distance can be extended to N_d-dimensional space for $N_d \geq 3$.

III. Analysis of Chaotic Map with Some slopes

In this section, we analyze chaotic maps for the chaotic sequence with biased values. To analyze the chaotic map in this study, we calculate the invariant density $\rho(x)$. $\rho(x)$ is the function deciding the iteration density of a map, namely we can observe the distribution of the value of the chaotic sequence. $\rho(x)$ is described by

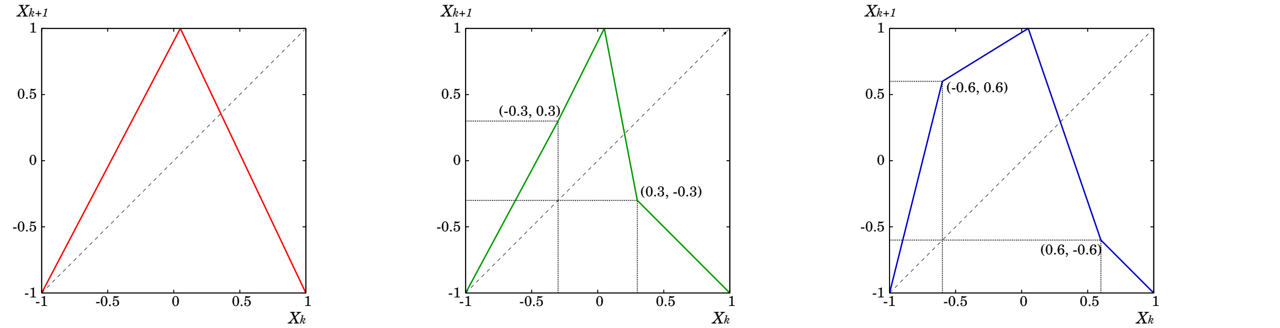
$$\rho(x) \equiv \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N} \delta(x - f^i(x_0)) \ , \tag{6}$$

where $x_{n+1} = f(x_n)$, $n = 0, \ 1, \ 2, \ \cdots$, x_0 is an initial value, δ is the delta function. If $\rho(x)$ is the system that is not dependent on an initial value, it is called ergodic. In this study, to calculate using the computer, N is assumed to 10^6.

We use three types of chaotic maps for calculating the invariant density; The first is the skew tent map as the chaotic map without biased values, as shown in Fig. 5(a); The second is the chaotic map with $(q_1, r_1) = (-0.3, 0.3)$, $(q_2, r_2) = (0.3, -0.3)$, as shown in Fig. 5(b); The third is the chaotic map with $(q_1, r_1) = (-0.6, 0.6)$, $(q_2, r_2) = (0.6, -0.6)$, as shown in Fig. 5(c). In this study, we label the second and the third as Type 1 and Type 2, respectively.

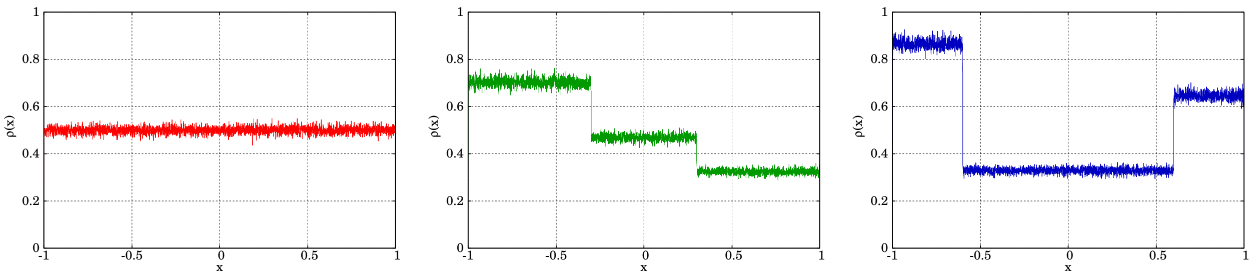
Figures 6(a), (b) and (c) show the invariant densities of the skew tent map, Type 1 and Type 2, respectively. First of all, we observe the invariant density of the skew tent map (Fig. 5(a)). From Fig 6(a), $\rho(x)$ is constant. In other words, the distribution of the chaotic sequence generated from the skew tent map is not dependent on an initial value, namely, we can confirm that the skew tent map is ergodic. Second, we focus on the invariant density of Type 1 (Fig 5(b)). As one can see, $\rho(x)$ increases as x approaches -1. Thus, the chaotic sequence generated from Type 1 includes many negative values. Finally, we observe the invariant density of Type 2 (Fig 5(c)). This distribution is divided into right and left with center on $x = 0$ according to (q_1, r_1). Especially, $\rho(x)$ increases as x approaches -1 or 1. Namely, the chaotic sequence generated from Type 2 includes many values around 1 and -1.

As described above, we become available the chaotic sequence with the desired biased values by changing the slope of the chaotic map.



(a) (b) (c)

Fig. 5. Chaotic maps: (a) Skew tent map, (b) Type 1 $(q_1, r_1) = (-0.3, 0.3)$, $(q_2, r_2) = (0.3, -0.3)$, (c) Type 2 $(q_1, r_1) = (-0.6, 0.6)$, $(q_2, r_2) = (0.6, -0.6)$.



(a) (b) (c)

Fig. 6. Invariant densities: (a) Skew tent map, (b) Type 1, (c) Type 2.

IV. Simulation Results

In this section, we carry out the computer simulation of our suboptimal receiver using Figs 5(a), (b) and (c). The simulation conditions are as follows. On the transmitting side, 10,000 symbols are transmitted using chaotic sequences with different initial values. Here, the parameter of the skew tent map is fixed as $a = 0.05$, As the chaotic sequence length, we set $N = 256$. On the receiving side, BER is recorded for various E_b/N_0. Also, to calculate the shortest distance, the receiver applies two-dimensional space (i.e., $N_d = 2$).

Figure 7 plots the BER versus E_b/N_0 for each chaotic map. To compare the performance, we perform the simulation of the skew tent map without biased values, i.e., as the conventional method. We can observe that the BER of Type 1 is the best performance in Fig 7. Since the chaotic sequence generated from Type 1 includes many negative values, the chaotic sequence with values which are biased toward one side might be suitable for our suboptimal receiver. From results, it can be said that the chaotic sequence which changed the distribution of biased values is effective for the suboptimal receiver. Therefore, chaotic sequences with biased values are effective not only DCSK but also the suboptimal receiver.

V. Conclusions

In this study, we have investigated the performance of the suboptimal receiver using chaotic sequences with biased values. As results, we have obtained the better BER performance by controlling the distribution of the chaotic sequence with biased values, Moreover, we have confirmed that the chaotic sequence with biased values is effective not only DCSK but also the suboptimal receiver.

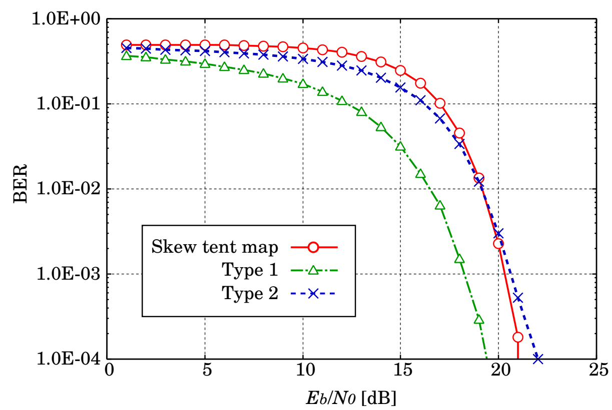


Fig. 7. BER performance ($N = 256$).

References

[1] G. Kolumbán, B. Vizvári, W. Schwarz, and A. Abel, "Differential chaos shift keying: A robust coding for chaos communication," *Proc. NDES'96*, pp. 87-92, Jun. 1996.

[2] M. Hasler and T. Schimming, "Chaos communication over noisy channels," *Int . Journal of Bifurcation and Chaos*, vol. 10, no. 4, pp. 719-736, Apr. 2000.

[3] F. C. M. Lau and C. K. Tse, *Chaos-Based Digital Communication Systems*, Springer, 2003.

[4] L. E. Larson, J-M. Liu, L. S. Tsimring, *Digital Communications Using Chaos and Nonlinear Dynamics*, Springer, 2006.

[5] G. Cimatti, R. Rovatti and G. Setti, "Chaos-Based Spreading in DS-UWB Sensor Networks Increases Available Bit Rate," *IEEE Trans. Circuits and Systems Part I*, vol. 54, no. 6, pp. 1327-1339, Jun. 2007.

[6] S. Arai and Y. Nishio, "Detection of Information Symbols and Sequence Lengths Using Suboptimal Receiver for Chaos Shift Keying," *Proc NOLTA'06*, pp. 799-802, Sep. 2006.

[7] S. Arai, Y. Nishio and T. Yamazato, "Analysis on Chaotic Sequence with Biased Values for Noncoherent Chaos Communication," *Proc NOLTA'07*, pp. 144-147, Sep. 2007.