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A Spice-Oriented Frequency Domain Peak Search Algorithm

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I. Introduction

In the peak search of the frequency characteristics, it is difficult to find the peak for large scale circuits. If we carry out frequency analysis as changing the frequency, we have to increase the frequency gradually. However, if the changing step is large, we may miss the peaks. Also there is no guarantee that the peak values exist at the frequencies.

In this article, we propose an algorithm to find the peak of the frequency characteristics by using the Sine-Cosine circuit [1]-[3] and nonlinear limiter to control the step size of SPICE.

II. Peak points tracing algorithm

Although, for relative low Q circuits, the AC analysis of Spice and a cubic spline combining Newton method can be usefully applied to find out the peak points, it may pass over the points for high Q circuits. Thus, we propose a new algorithm based on the HB method such that the determining equation is solved by transient analysis of Spice, where ω is a function of time t as follows:

$$\omega = Kt \tag{1}$$

Now, let us discuss the Sine-Cosine circuit corresponding to the determining equation of the HB method.

Let the current through an inductance L be

$$i_L = I_{LS}\sin\omega t + I_{LC}\cos\omega t \tag{2.1}$$

Then, the voltage v_L is given by

$$v_L = L\frac{di_L}{dt} = -\omega L I_{LC}\sin\omega t + \omega L I_{LS}\cos\omega t \tag{2.2}$$

Thus, the coefficients of $\sin\omega t$, $\cos\omega t$ are described by

$$V_{LS} = -\omega L I_{LC}, \quad V_{LC} = \omega L I_{LS} \qquad (2,3)$$

Namely, the inductance is replaced by coupled current-controlled voltage sources in the Sine-Cosine transformation of the HB method. In the same way, let the voltage across a capacitor C be

$$v_C = V_{CS}\sin\omega t + V_{CC}\cos\omega t \tag{3.1}$$

Then, the current i_C is given by

$$i_C = C\frac{dv_C}{dt} = -\omega C V_{LC}\sin\omega t + \omega C V_{LS}\cos\omega t \tag{3.2}$$

Thus, the coefficients of $\sin\omega t$, $\cos\omega t$ are described by

$$I_{LS} = -\omega C V_{LC}, \quad I_{LC} = \omega C V_{LS} \qquad (3,3)$$

Namely, the capacitor is replaced by the coupled voltage-controlled current sources in the Sine-Cosine transformation. The circuit corresponding to the determining equation of the HB method is driven by a constant voltage $E = J$ and ω is given by (1).

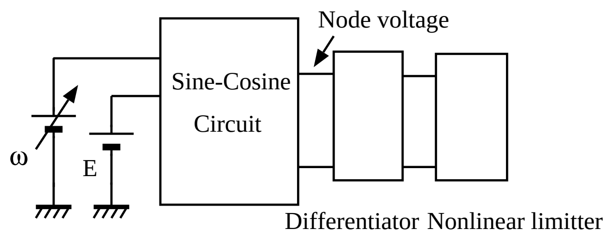


Fig. 1. Peak detector of frequency response curve.

Since ωs at the peak voltages satisfy

$$\frac{d|V_k(\omega)|}{d\omega} = 0, \quad k = 1, 2, \ldots n, \tag{4}$$

on the response curve, we need to find the zero points satisfying (4) Hence, $|V_k(\omega)|$ need to be firstly differentiated by a differentiator. In order to detect the exact peak points, the output is limited and expanded with a nonlinear limiter as shown in Fig. 1, which consists of a limiter and nonlinear diodes. The output of the limiter is given by

$$v_L = \begin{cases} -V_{max} & : \quad \text{for} \;\; v_{in} < -V_L \\ k v_{in} & : \quad \text{for} \;\; -V_L \geq v_{in} \geq V_L \\ V_{max} & : \quad \text{for} \;\; v_{in} > V_L \end{cases} \tag{5.1}$$

where

$$kV_L = V_{max} \tag{5.2}$$

In our example of section 3, we set $V_L = 2[\mu V]$, $V_{max} = 100[V]$, and $k = 0.5 \times 10^8$. The outputs of diodes are

$$i_o = \begin{cases} I_s \exp(\lambda v_o) & : \quad \text{for} \quad v_o > 0 \\ -I_s \exp(-\lambda v_o) & : \quad \text{for} \quad v_o < 0 \end{cases} \tag{5.3}$$

for $I_s = 10^{-12}$ and $\lambda = 40$.

The characteristic of the nonlinear limiter is shown in Fig. 2. This means that the regions around $d|V_k(\omega)|/d\omega = 0$ are

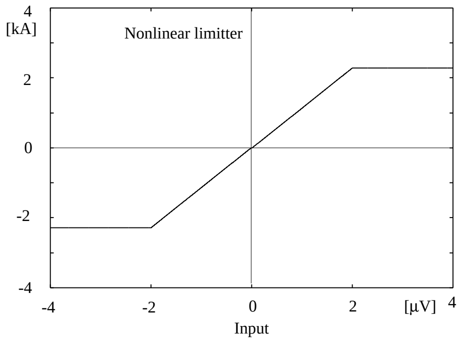


Fig. 2. Input and output characteristic of nonlinear limiter.

largely expanded. Furthermore, the characteristic has large nonlinearity around the zeros. Thus, the transient analysis around the zero points is executed with a very small step size, and we can find out precise peak points.

III. Illustrative example

Transmission line is usually modeled by discrete RLCG ladder circuit as shown in Fig. 3, where we neglected G. This time, we analyzed the cases of the numbers of the ladder are 4 and 8.

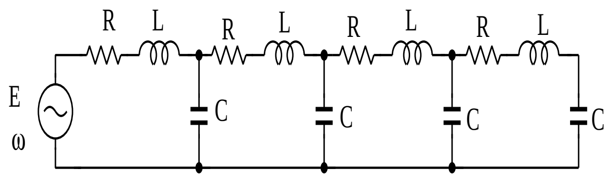


Fig. 3. LRC ladder circuit(ladder is 4) $C = 1[nF]$, $L = 10[mH]$, $e(t) = E \cos \omega t$ and $E = 1[\mu V]$.

In order to obtain the frequency response curve, the circuit is transformed by HB method into the Sine-Cosine circuit. The peak detector given in section 2 is attached as shown in Fig. 4. Note that the detector can be attached to an arbitrary node.

Now, we show the simulation results using the transient analysis of Spice. The circuit in case of ladder is 4 which has 4 resonant points as shown by Figs. 5(a) and (b) which show the voltages at the far end of the circuit in Fig. 3, where we set the peaks 1,2,3,4 from the left hand side. In the same way, the circuit in case of ladder is 8 which has 8 resonant points as shown by Figs. 6(a) and (b) which show the voltages at the far end of the circuit, where we set from the peaks 1 to the

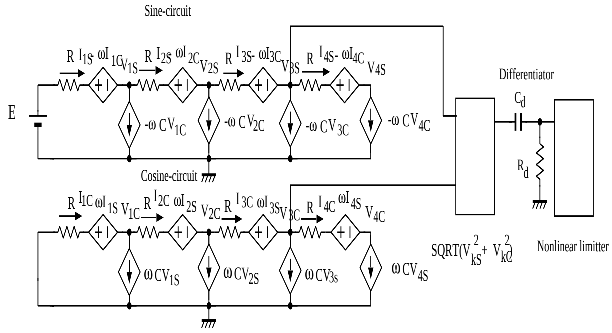
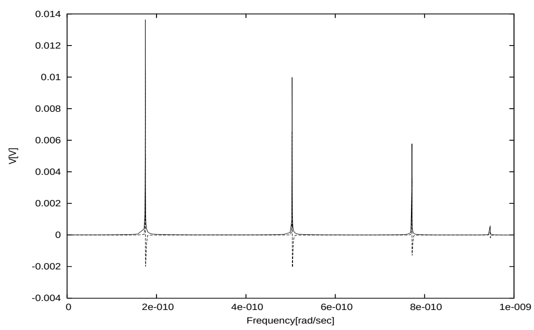
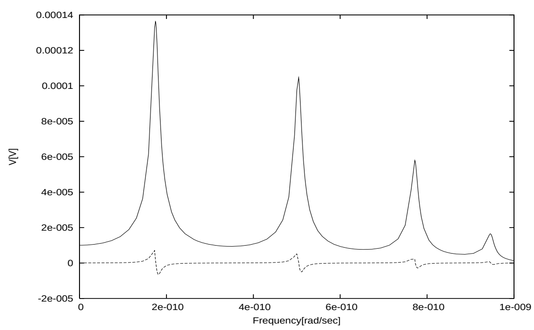


Fig. 4. Sine-Cosine circuit combining with peak detector. $C_d = 1[pF]$ and $R_d = 1[\Omega]$.

peaks 8 from the left hand side. The sharpnesses depend on the quality factors so that we have changed them by Rs. We found that although the resonant frequencies are almost same for all Rs as shown in Table 1 and table 2, the peak values are largely different.

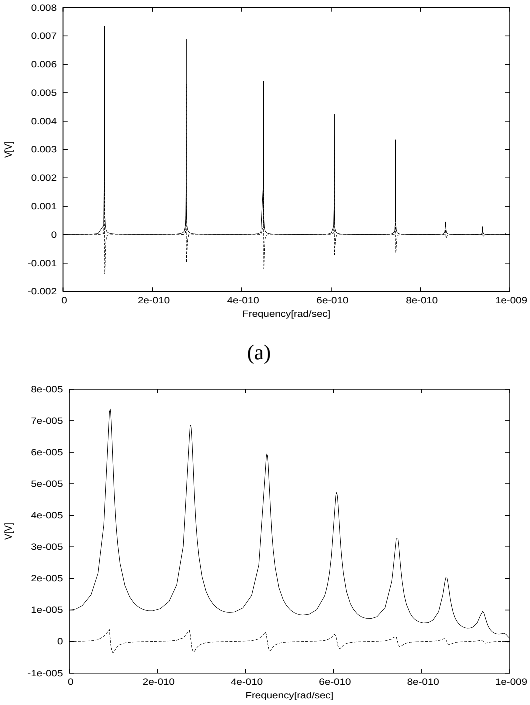


(a)



(b)

Fig. 5. Frequency characteristics of the circuit in Fig. 4 (ladder is 4). (a) R=1[Ω]. (b) R=100[Ω].



(a)

(b)

Fig. 6. Frequency characteristics of the circuit (ladder is 8). (a) R=1[Ω]. (b) R=100[Ω].

Table 1 Calculated peak values (ladder is 4).

R=1[Ω]	peak 1	peak 2	peak 3	peak 4
ω[prad/sec]	175.1	503.7	771.6	946.7
V[mV]	13.6	10.0	5.8	0.5
R=100[Ω]	peak 1	peak 2	peak 3	peak 4
ω[prad/sec]	175.4	504.1	772.2	944.8
V[μV]	136.2	104.6	57.9	16.5

Table 2 Calculated peak values (ladder is 8).

R=1[Ω]	peak 1	peak 2	peak 3	peak 4
ω[prad/sec]	93.1	275.8	449.1	607.1
V[mV]	7.3	6.9	5.4	4.2
-	peak 5	peak 6	peak 7	peak 8
ω[prad/sec]	744.4	856.8	939.5	990.7
V[mV]	3.3	0.5	0.3	0.04
R=100[Ω]	peak 1	peak 2	peak 3	peak 4
ω[prad/sec]	93.1	276.1	449.5	606.4
V[μV]	73.5	68.6	59.4	47.2
-	peak 5	peak 6	peak 7	peak 8
ω[prad/sec]	741.5	854.4	937.0	980.2
V[μV]	32.8	20.2	9.6	2.6

It seems that the calculated peak 4 in table 1 and peak 8 in table 2 for $R = 1$ are a little bit larger than the exact value. It is due to pass over the point because the peak is too narrow and small to detect even when the nonlinear limiter is introduced. In this case, we need to trace the resonant curve around the peak 4 and peak 8 with a selection of much smaller initial step size.

IV. CONCLUSIONS AND REMARKS

In this study, we have proposed an algorithm to find the peaks of the frequency response curve by combining the Sine-Cosine circuit based on the HB method. The circuit is traced by the transient analysis of Spice. Since the peak points correspond to the gradient being to zeros, we differentiate the curve and find its zero points. In order to find the exact zero points, we have developed the peak detector using nonlinear limiter such that the regions are largely expanded. Thus, the step size around zero values becomes small when we use variable step size transient analysis.

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