Particle Swarm Optimization with Map Structure

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Abstract—This study proposes a Particle Swarm Optimization with Map Structure (PSOMS). All particles of PSOMS are connected to adjacent particles by neighborhood relation, which dictates the topology, of the 2-dimensional map. Each particle is updated depending on the neighborhood distance between it and a winner, whose function value is best among all particles. Simulation results show the searching efficiency of PSOMS.

I. INTRODUCTION

Particle Swarm Optimization (PSO) [1] is an evolutionary algorithm to simulate the movement of flocks of birds. Due to the simple concept, easy implementation, and quick convergence, PSO has attracted much attention and is used to wide applications in different fields in recent years. However, PSO greatly depends on its parameters and converge prematurely in case of solving complex problems which have local optima. Furthermore, in PSO algorithm, there are no special relationships between particles. Each particle position is updated according to its personal best position and the best particle position among all particles, and their weights are determined at random in every generation. On the other side, in the real world, various personal relationships exist, such as the hierarchical relationship, the trust relationships, the parent-child relationship and so on.

In this study, we propose a new Particle Swarm Optimization with Map Structure (PSOMS). All particles of PSOMS are connected to adjacent particles by neighborhood relation, which dictates the topology, of the 2-dimensional map. In every generation, we find a winner particle, whose function value is best among all particles, and each particle is updated depending on the neighborhood distance between it and the winner on the map. Simulation results and comparisons with the standard PSO show that the proposed PSOMS can effectively enhance the searching efficiency.

II. PARTICLE SWARM OPTIMIZATION WITH MAP STRUCTURE (PSOMS)

In the algorithm of PSO, multiple solutions called “particles” coexist. The most important feature of PSOMS is that all particles are organized on a rectangular 2-dimensional grid. In other words, the particles are connected to adjacent particles by neighborhood relation, which dictates the topology, of the map. The position vector of each particle’s velocity vector are represented by \( X_i = (x_{i1}, \ldots, x_{id}, \ldots, x_{id}) \) and \( V_i = (v_{i1}, \ldots, v_{id}, \ldots, v_{id}) \), respectively, where \((d = 1, 2, \ldots, D), (i = 1, 2, \ldots, M)\) and \( x_{id} \in [x_{\text{min}}, x_{\text{max}}] \).

(PSOMS1) (Initialization) Let a generation step \( t = 0 \). Randomly initialize the particle position \( X_i \) and its velocity \( V_i \) for all particles \( i \), and initialize \( P_i = (p_{i1}, p_{i2}, \ldots, p_{iD}) \) with a copy of \( X_i \). Evaluate the objective function \( f(X_i) \) for all particle \( i \) and find \( P_g \) with the best function value among the all particles. Define \( g \) as the winner c.

(PSOMS2) Evaluate the fitness \( f(X_i) \) and find the winner particle \( c \) with the best fitness among all the particles at current time.

\[
c = \arg\min \{ f(X_i(t)) \}.
\]

For each particle \( i \), if \( f(X_i(t)) < f(P_i) \), the personal best position (called pbest) \( P_i = X_i \).

Let \( P_g \) represents the best position with the best fitness among all particles so far (called gbest). If \( f(X_c) < f(P_g) \), update gbest \( P_g = X_c \), where \( X_c \) is the position of the winner \( c \).

(PSOMS3) Update \( V_i \) and \( X_i \) of each particle \( i \) depending on its pbest, gbest and the distance on the map between \( i \) and the winner \( c \), according to

\[
v_{id}(t + 1) = w v_{id}(t) + c_1 \text{rand}(\cdot) (p_{id} - x_{id}(t)) + c_2 h_{c,j} (x_{cd} - x_{id}(t)),
\]

\[
x_{id}(t + 1) = x_{id}(t) + v_{id}(t + 1),
\]

where \( w \) is the inertia weight determining how much of the previous velocity of the particle is preserved. \( c_1 \) and \( c_2 \) are two positive acceleration coefficients, generally \( c_1 = c_2 \). \text{rand}(\cdot) is an uniform random numbers samples from \( U(0,1) \). \( h_{c,i} \) is the fixed neighborhood function defined by

\[
h_{c,i} = \exp \left( -\frac{\| r_i - r_c \|^2}{2\sigma^2} \right),
\]

where \( \| r_i - r_c \| \) is the distance between map nodes \( c \) and \( i \) on the map, the fixed parameter \( \sigma \) corresponds to the width of the neighborhood function. Therefore, the large \( \sigma \) strengthens particles’ spreading force to the whole space, and the small \( \sigma \) strengthens their convergent force toward the winner.

(PSOMS4) Let \( t = t + 1 \) and go back to (PSOMS2).

III. EXPERIMENTAL RESULTS

In order to evaluate the performance of PSOMS, we use two benchmark optimization problems. One is the Rosenbrock function \( f_1 \) as Eq. (4) and the other is the Rastrigin function
TABLE I
COMPARISON RESULTS OF PSO AND PSOMS FOR $f_1$ AND $f_2$.

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>PSO</td>
<td>178.559</td>
<td>95.0156</td>
</tr>
<tr>
<td></td>
<td>PSOMS</td>
<td>102.9688</td>
<td>96.4675</td>
</tr>
<tr>
<td>$f_2$</td>
<td>PSO</td>
<td>444.3451</td>
<td>337.3337</td>
</tr>
<tr>
<td></td>
<td>PSOMS</td>
<td>178.9360</td>
<td>114.3723</td>
</tr>
</tbody>
</table>

Fig. 1. Mean Rosenbrock function value of every generation.

Fig. 2. Mean Rastrigin function value of every generation.

We carry out the simulations repeated 30 times for each optimization function with 2000 time steps. The performance of PSO and PSOMS with their corresponding minimum and mean function values are listed in Table I. Figures 1 and 2 show the mean value of the best function value of every generation over 30 runs for $f_1$ and $f_2$ function, respectively. From these results, we can see that the results of PSOMS have better accuracy. In PSO, the number of particles which move toward $gbest$ or toward $pbest$ is decided by random on every generation and is not stable. On the other hand, the neighborhood gaussian function is used in PSOMS, therefore, the particles move according to the neighborhood distance between the winner and them. The winner’s neighborhood particles move beyond the winner so that they spread to whole space. The particles, which are connected at a little distance from the winner, move toward the winner. The other particles fly toward their $pbest$. In other words, the roles of the PSOMS particles are determined by the connection relationship.

Furthermore, in order to investigate the effect of the parameters; the inertia weight $w$ and the acceleration coefficients $c_1$ and $c_2$, on performance quality and their sensitivity, Figs. 3 and 4 show the mean function values with different parameters. The fixed parameters are same as above simulations. The proposed PSOMS is more effective and the parametrical dependence is not stronger than PSO. The performance of PSO is sensitive to the parameters, however, the performance of PSOMS is stable.

IV. CONCLUSIONS

This study has proposed a Particle Swarm Optimization with Map Structure (PSOMS). All particles of PSOMS are connected to adjacent particles by neighborhood relation, which dictates the topology, of the 2-dimensional map. Each particle is updated depending on the neighborhood distance between it...
and a winner, whose function value is best among all particles. In the simulation results, the searching efficiency of PSOMS is better than PSO. Furthermore, we have confirmed that the parametrical dependence of PSOMS is not stronger than PSO.

REFERENCES