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Synchronization Phenomena in Three Oscillators Coupled by a Resonator

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1. Introduction

There have been many investigations of the mutual synchronization of oscillators [1]. Moro et al. confirmed that N oscillators with same natural frequencies mutually coupled by one resistor give N-phase oscillations, when N is a prime number. In this study, we observe synchronization of 3 oscillators coupled by a resonator.

2. Circuit Model

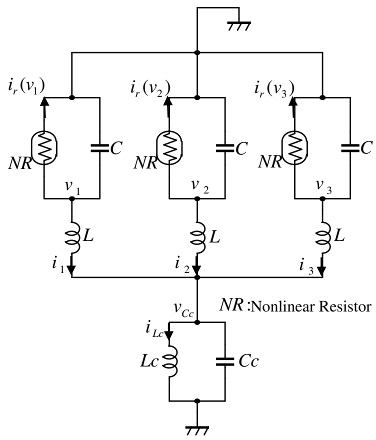

Figure 1: Circuit model (3 oscillators case).

The circuit model is shown in Fig. 1. The circuit equations are described as follows,

$$\begin{cases} C\dfrac{dv_k}{dt} = -i_k - i_r(v_k) \\ L\dfrac{di_k}{dt} = v_k - v_{Cc} \qquad (k=1,2,3) \\ Cc\dfrac{dv_{Cc}}{dt} = \displaystyle\sum_{j=1}^{3} i_j - i_{Lc} \\ Lc\dfrac{di_{Lc}}{dt} = v_{Cc} \end{cases} \tag{1}$$

where $i_r(v_k)$ indicates the $v-i$ characteristics of the nonlinear resistor and is approximated by the following function.

$$i_r(v_k) = -g_1 v_k + g_3 v_k^3. \tag{2}$$

By using the following variables and parameters,

$$\begin{cases} v_k = \sqrt{\dfrac{g_1}{g_3}}x_k, \quad i_k = \sqrt{\dfrac{Cg_1}{Lg_3}}y_k \\ v_{Cc} = \sqrt{\dfrac{g_1}{g_3}}X, \quad i_{Lc} = \sqrt{\dfrac{Cg_1}{Lg_3}}Y \\ t = \sqrt{LC}\tau, \quad \text{“}\cdot\text{”} = \dfrac{d}{d\tau} \\ \epsilon = \sqrt{\dfrac{L}{C}}g_1, \quad \beta = \dfrac{C}{Cc}, \quad \gamma = \dfrac{L}{Lc} \end{cases} \tag{3}$$

the normalized circuit equations are given as follows.

$$\begin{cases} \dot{x}_k = -y_k + \epsilon(x_k - x_k^3) \\ \dot{y}_k = x_k - X \qquad (k=1,2,3) \\ \dot{X} = \beta\left(\displaystyle\sum_{j=1}^{3} y_j - Y\right) \\ \dot{Y} = \gamma X \end{cases} \tag{4}$$

3. Synchronization Phenomena

As showing in Figs. 2(a)(b)(c), we can observe two patterns of oscillations in the same parameter, 3-phase oscillation and in-phase oscillation. When the parameters are changed, the three circuits synchronized in another type of in-phase oscillation as shown in Fig. (d). It is interesting that the oscillation frequency depends on the synchronization patterns.

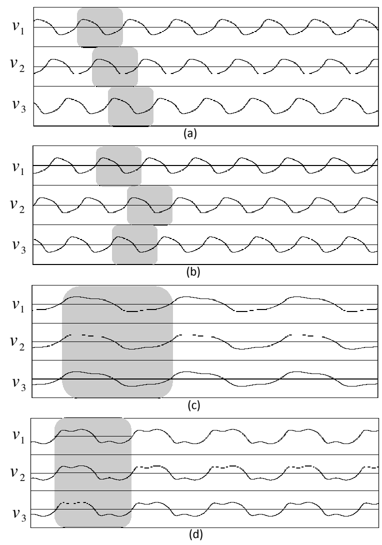

Figure 2: Time waveform of 3-phase oscillation and in-phase oscillation (numerical results). (a)(b)(c) $\epsilon = \beta = \gamma = 1.0$. (d) $\epsilon = 1.0,\ \beta = 0.5,\ \gamma = 2.0$.

Also, the circuit experimental results show similar phenomenon to the numerical results.

4. Conclusions

In this article, we presented synchronization phenomena of 3 oscillators coupled by a resonator. Our future work is to investigate these synchronization phenomena in more detail.

References

[1] Seiichiro MORO, Yoshifumi NISHIO and Shinsaku MORI, “Synchronization Phenomena in Oscillators Coupled by One Resistor,” IEICE Transactions on Fundamentals, vol. E78-A, no. 2, pp. 244-253, Feb. 1995.