17-8

Breakdown of Chaos Synchronization and Noise Effect on Simple Oscillators

Ryo Imabayashi **Yoko Uwate** **Yoshifumi Nishio**
(Tokushima University)

1 Introduction

In our previous research, we have investigated the breakdown of synchronization observed from four coupled chaotic oscillators. In order to understand the phenomenon, the model of coupled modified van der Pol oscillators with the additive white Gaussian noise (AWGN) was proposed. By computer simulations, we have confirmed that chaotic systems were synchronized more stably than modified van der Pol oscillators with AWGN.

In this study, we propose other two types of noise for adding the van der Pol oscillators to confirm the different between chaotic systems and van der Pol systems with AWGN. First noise has two band characteristic, and second noise is a scaled AWGN. By adding the scaled AWGN, modified van der Pol oscillators are close to coupled chaotic systems is confirmed.

2 Circuit Model

Fig. 1 shows the circuit model. In the circuit, four identical chaotic circuits are coupled by one resistor.

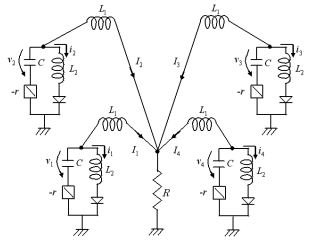


Figure 1: Coupled chaotic oscillators.

The normalized circuit equations of the circuit are described as

$$\begin{cases} \dot{x}_k = \beta(x_k + y_k) - z_k - \gamma \sum_{j=1}^{4} x_j \\ \dot{y}_k = \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\} \\ \dot{z}_k = x_k + y_k, \qquad (k = 1, 2, 3, 4) \end{cases} \tag{1}$$

where $f(y_k) = 0.5(\delta\ y_k + 1 - |\ \delta\ y_k - 1|)$.

Next, we consider four coupled modified van der Pol oscillators. In order to obtain the waveforms similar to those of the chaotic oscillator, we modify the van der Pol oscillator with the nonlinear resistor whose $v - i$ characteristics are described by the following asymmetric function

$$i_r(v_k) = -g_1 v_k + g_2 v_k^2 + g_3 v_k^3 \quad (g_1, g_2, g_3 > 0). \tag{2}$$

When we add the noise to the voltage amplitude of the modified van der Pol oscillator, the circuit equation of the coupled oscillators are described as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \xi[-y_k + \varepsilon\{(1 + \rho_k n_k(\tau))x_k \\ \qquad - \nu((1 + \rho_k n_k(\tau))x_k)^2 - ((1 + \rho_k n_k(\tau))x_k)^3\}] \\ \dfrac{dy_k}{d\tau} = (1 + \rho_k n_k(\tau))x_k - \gamma \sum_{j=1}^{4} y_j, \qquad (k = 1, 2, 3, 4) \end{cases} \tag{3}$$

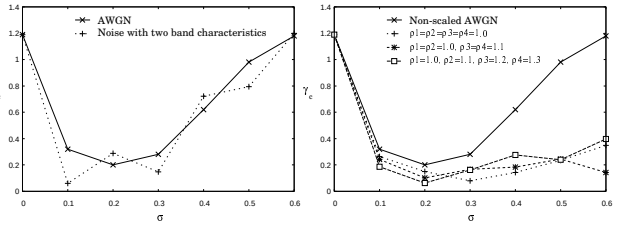
While when we add the noise to the voltage period of the modified van der Pol oscillator, the circuit equation of the coupled oscillators are described as

$$\begin{cases} \dfrac{dx_k}{d\tau} = (1 + \rho_k n_k(\tau))\xi\{-y_k + \varepsilon(x_k - \nu x_k^2 - x_k^3)\} \\ \dfrac{dy_k}{d\tau} = x_k - \gamma \sum_{j=1}^{4} y_j, \qquad (k = 1, 2, 3, 4) \end{cases} \tag{4}$$

where $n_k(\tau)$ is the added noise and ρ_k is constant to tune the amplitude of the noise. The noise $n_k(\tau)$ is the additive white Gaussian noise (AWGN) with the average 0 and the variance σ^2.

3 Computer Calculated Results

When the coupling parameter γ is relatively large, both the coupled chaotic oscillators and the modified van der Pol oscillators with noise exhibit four phase synchronizations. While for relatively smaller γ, the synchronizations break down and we observe the switchings of phase states. We define this critical coupling parameter as γ_c and investigate how γ_c changes when the strength of chaos or noise increases.



(a) (b)

Figure 2: Breakdown of synchronization of modified van der Pol oscillators with noisy amplitude ($\epsilon = 0.5$, $\xi = 1.07$, $\nu = 0.3$, $\mu = 2.795$ and $\sigma = 1.0$). (a) Noise with two band characteristics ($\rho_k = 1.0$). (b) Scaled AWGN.

Fig. 2 (a) show synchronization of modified van der Pol oscillators with noise with two band characteristics. From these results, the breakdown of synchronization is lightly affected by the distribution of noise.

It is seen that non-scaled AWGN has a smaller area of stable synchronization in Fig. 2 (b). From these results, we can say that by adding the scaled AWGN the modified van der Pol oscillators behave more similar to the coupled chaotic systems.

4 Conclusions

In this study, the breakdown of synchronization observed from four coupled chaotic oscillators has been investigated. In order to understand the phenomenon, the model of coupled modified van der Pol oscillators with noise was considered. By adding the scaled AWGN, the modified van der Pol oscillators are confirmed to be closer to the coupled chaotic circuits.

References

[1] R. Imabayashi, Y. Uwate and Y. Nishio, "Breakdown of Synchronization in Chaotic Oscillators and Noisy Oscillators," *Proceedings of European Conference on Circuit Theory and Design (ECCTD'07)*, pp. 922-925, 2007.