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Synchronization Phenomena in Four Coupled Parametrically Excited van der Pol Oscillators

Hironori KUMENO (Tokushima University)
Yoshifumi NISHIO (Tokushima University)

1. Introduction

Studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics, biology, engineering and so on. We consider that it is important to investigate the synchronization phenomena of coupled oscillators for the future engineering application. In a past study, we investigated parametrically excited van der Pol oscillators coupled by a resister. And we obtained various kinds of synchronization phenomena in the case of two or three coupled subcircuit. In this study, we investigate synchronization phenomena in four coupled parametrically excited van der Pol oscillators.

2. van der Pol oscillator under parametric excitation

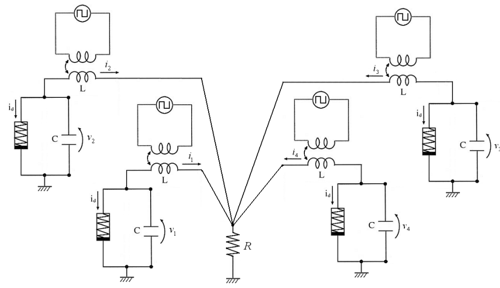

Figure 1: Circuit model.

The circuit model used in this study is shown in Fig. 1. The circuit includes a time-varying inductor L whose characteristics are given as the following equation.

$$L = L_0\gamma(t). \tag{1}$$

$\gamma(\tau)$ is expressed in a rectangular wave as shown in Fig. 2, and its amplitude and angular frequency are termed α and ω, respectively.

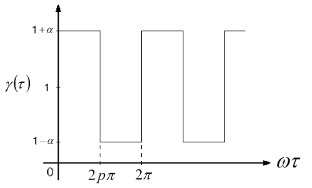

Figure 2: Function relating to parametrically excitation.

The $v-i$ characteristics of the nonlinear resistor are approximated by the following equation.

$$i_d = -g_1 v_k + g_3 v_k. \tag{2}$$

By changing the variables and the parameters,

$$\begin{aligned} &t = \sqrt{L_0 C}\tau, \quad v_k = \sqrt{\frac{g_1}{g_3}}x_k, \quad \delta = \sqrt{\frac{C}{L_0}}R, \\ &i_k = \sqrt{\frac{g_1}{g_3}}\sqrt{\frac{C}{L_0}}y_k, \quad \varepsilon = g_1\sqrt{\frac{L_0}{C}}, \end{aligned} \tag{3}$$

the normalized circuit equations are given by the following equations.

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon(x_k - x_k^3) - y_k \\ \dfrac{dy_k}{d\tau} = \dfrac{1}{\gamma(\tau)}x_k - \delta\displaystyle\sum_{j=1}^{4} y_j. \end{cases} \tag{4}$$

When parameter ε changes, periodic attractors, quasi-periodic attractors and chaotic attractors are confirmed to be generated in the isolated subcircuit. Figure 3 shows an example of chaotic attractors and its Poincaré map observed from the isolated subcircuit. We define the Poincaré section as $\omega\tau = 2n\pi$.

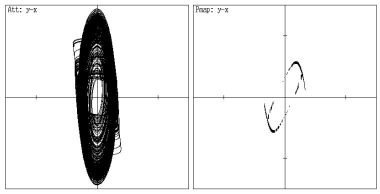

Figure 3: Example of chaotic attractors and its Poincaré map observed from subcircuit. $\alpha = 0.95$, $\omega = 1.50$ and $\varepsilon = 1.5$.

3. Simulation result

We carry out computer calculations for four subcircuits. In this case, we observed two different types of synchronization phenomena; in and opposite-phases synchronization and self-switching of two-pairs opposite-phase synchronizations (see Fig. 4). These two types of synchronizations were observed for different initial values.

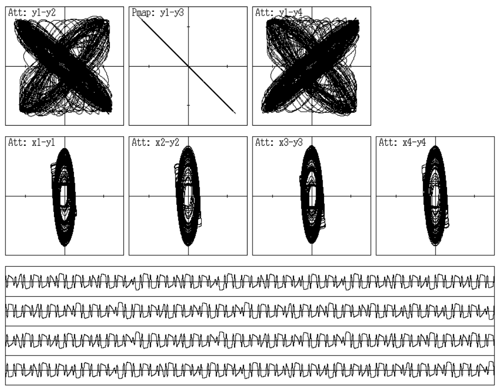

Figure 4: Self-switching of two-pairs opposite-phase synchronizations. $\varepsilon = 1.50$, $\alpha = 0.95$, $\omega = 1.50$ and $\delta = 5.00$.

4. Conclusions

In this study, we investigated synchronization of parametrically excited van der Pol oscillators. By carrying out computer calculations for four subcircuits, we confirmed that various kinds of synchronization phenomena of chaos were observed; in and opposite-phases synchronization and self-switching of two-pairs opposite-phase synchronizations.