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Synchronization Patterns in Three Cross-Coupled Chaotic Circuits

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1. Introduction

Studies on synchronization phenomena of coupled chaotic circuits are extensively carried out in various fields.

In our past studies, we investigated the state transition phenomenon in two cross-coupled chaotic circuits. This state transition phenomena can be observed around in-phase synchronization, anti-phase synchronization and quadrature-phase synchronization.

In this study, synchronization patterns generated in three cross-coupled chaotic circuits are investigated. Computer simulations and circuit experiments show that this system produces several phase patterns.

2. Circuit Model

Figure 1 shows the circuit model. In the circuit, three chaotic circuits are cross-coupled via inductors and small resistors.

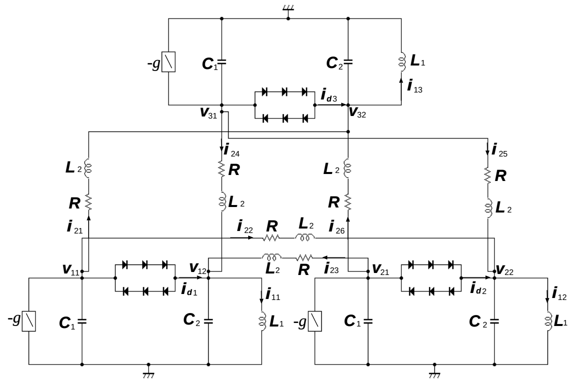

Figure 1: Circuit model.

By using the following variables and the parameters,

$$\begin{cases} x_k = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{k1}}{V}, \quad w_k = \sqrt{\dfrac{L_1}{C_2}}\,\dfrac{i_{k2}}{V}, \quad y_k = \dfrac{v_{k1}}{V}, \\ z_k = \dfrac{v_{k2}}{V}, \quad t = \sqrt{L_1 C_2}\,\tau, \quad \alpha = \dfrac{C_2}{C_1}, \quad \beta = \sqrt{\dfrac{L_1}{C_2}}\,G, \\ \gamma = \sqrt{\dfrac{L_1}{C_2}}\,g, \quad \delta = \dfrac{L_1}{L_2}, \quad \epsilon = \sqrt{\dfrac{C_2}{L_1}}\,R, \quad \text{“}\cdot\text{”} = \dfrac{d}{d\tau} \end{cases} \tag{1}$$

the normalized circuit equations are given as follows.

$$\begin{cases} \dot{x}_k = z_k \\ \dot{y}_1 = \alpha\{\gamma y_1 - w_1 - w_2 - \beta f(y_1 - z_1)\} \\ \dot{y}_2 = \alpha\{\gamma y_2 - w_3 - w_6 - \beta f(y_2 - z_2)\} \\ \dot{y}_3 = \alpha\{\gamma y_3 - w_4 - w_5 - \beta f(y_3 - z_3)\} \\ \dot{z}_1 = \beta f(y_1 - z_1) + w_4 + w_3 - x_1 \\ \dot{z}_2 = \beta f(y_2 - z_2) + w_2 + w_5 - x_2 \\ \dot{z}_3 = \beta f(y_3 - z_3) + w_1 + w_6 - x_3 \\ \dot{w}_1 = \delta(y_1 - z_3 - \epsilon w_1), \quad \dot{w}_2 = \delta(y_1 - z_2 - \epsilon w_2) \\ \dot{w}_3 = \delta(y_2 - z_1 - \epsilon w_3), \quad \dot{w}_4 = \delta(y_3 - z_1 - \epsilon w_4) \\ \dot{w}_5 = \delta(y_3 - z_2 - \epsilon w_5), \quad \dot{w}_6 = \delta(y_2 - z_3 - \epsilon w_6) \end{cases} \tag{2}$$

where f are nonlinear functions corresponding to the $v-i$ characteristics of the nonlinear resistors and are described as follows.

$$f(y_k - z_k) = \begin{cases} y_k - z_k - 1 & (y_k - z_k > 1) \\ 0 & (|y_k - z_k| \leq 1) \\ y_k - z_k + 1 & (y_k - z_k < -1) \end{cases} \tag{3}$$

3. Synchronization Patterns

The synchronization states can be expressed by the phase differences of y_2 and y_3 with respect to the reference waveform y_1. For example, fully in-phase synchronization can be written as $[0, 0, 0]$. Also, if y_2 is synchronized to y_1 with $\pi/2$ phase difference (quadrature-phase) and y_3 is synchronized to y_1 with π phase difference (anti-phase), the state can be written as $[0, \pi/2, \pi]$. By using this notation, all possible combinations of the phase states can be summarized as follows;

$$\begin{array}{ll} \text{TYPE I}: & [0, 0, 0] \\ \text{TYPE II}: & [0, \pi/2, \pi/2] \\ \text{TYPE III}: & [0, \pi, \pi] \\ \text{TYPE IV}: & [0, \pi/2, \pi] \end{array} \tag{4}$$

All other combinations of phase states can be obtained from these type by using the symmetry of the coupling structure.

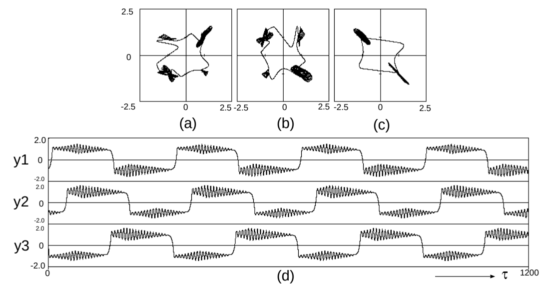

Figure 2: Synchronization states (TYPE IV). $\alpha = 2.5$, $\beta = 4.0$, $\gamma = 0.1$, $\delta = 0.0007$, and $\epsilon = 0.0005$. (a) Attractor on $y_1 - y_2$ plane. (b) Attractor on $y_2 - y_3$ plane. (c) Attractor on $y_3 - y_1$ plane. (d) Time waveform.

Figure 2 shows computer calculated results of one example of the all possible types of synchronization states observed from the circuit in Fig 1. Also, the circuit experimental results show the similar phenomenon to the computer calculated results.

4 Conclusions

In this study, we have investigate the synchronization patterns characterized by the synchronization states in three cross-coupled chaotic circuits. We confirmed that several patterns could be observed by giving different initial conditions to the circuits.