Synchronization in a Ring of van der Pol Oscillators with Different Frequencies

Yoko Uwate (Tokushima University)　Yoshifumi Nishio (Tokushima University)　Ruedi Stoop (UNI / ETH Zurich)

1. Introduction

In this study, we investigate synchronization phenomena in a ring of van der Pol oscillators with different frequencies. Namely, only the Nth oscillator has different frequency. By carrying out computer simulations, we observe that oscillation of the Nth oscillator stops in some range of the frequency ratio and that others are synchronized as if the Nth oscillator does not exist.

2. Circuit Model

The circuit model is shown in Fig. 1. In the system only the Nth oscillator has different oscillation frequency.

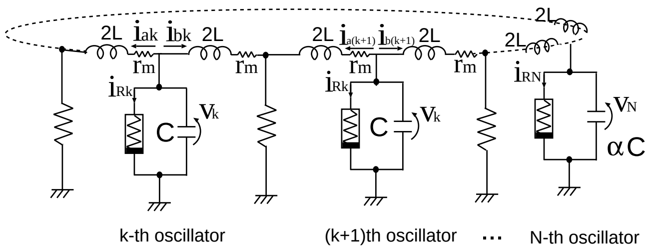


Fig. 1: Circuit model.

The normalized circuit equations of the array of oscillators are given as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \varepsilon(1 - {x_k}^2) - (y_{ak} + y_{bk}) \\ \dfrac{dx_N}{d\tau} = \omega^2\varepsilon(1 - {x_N}^2) - \omega^2(y_{aN} + y_{bN}) \\ \dfrac{dy_{ak}}{d\tau} = \dfrac{1}{2}x_k - \eta y_{ak} - \gamma(y_{ak)} + y_{b(k+1)}) \\ \dfrac{dy_{bk}}{d\tau} = \dfrac{1}{2}x_k - \eta y_{bk} - \gamma(y_{a(k-1)} + y_{bk)}) \\ \qquad (k = 1, 2, \cdots, N) \end{cases} \quad (1)$$

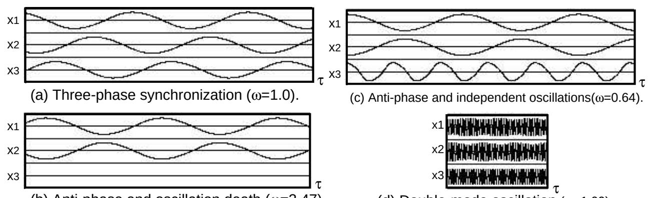
where

$$y_{a0} = y_{aN}, \qquad y_{b(N+1)} = y_{b1}. \quad (2)$$

It should be noted that γ corresponds to the coupling strength and that ε corresponds to the nonlinearity of oscillators.

3. Synchronization Phenomena

Figure 2 shows observed phenomena for $N = 3$. For the case that all of these oscillations have the same frequency, we can observe that system is synchronized at the three-phase (Fig. 2(a)). When the frequency of the 3rd oscillator is varied, we confirm that oscillation of the 3rd oscillator stops, namely oscillation death appears (Fig. 2(b)). As increasing the frequency of the 3rd oscillator, oscillation of the 3rd oscillator starts again (Fig. 2(c)). However, the 3rd oscillator is not synchronized to the others. Namely, the 3rd oscillator oscillates alone and the others keep anti-phase synchronization. Furthermore, we observe the double-mode oscillation as shown in Fig. 2(d).

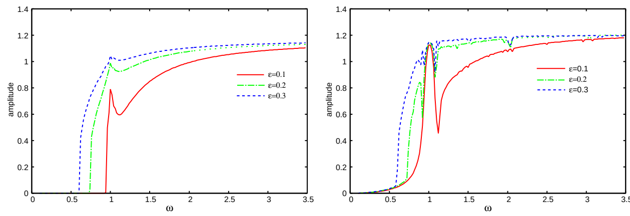


(a) Three-phase synchronization (ω=1.0). (b) Anti-phase and oscillation death (ω=2.47). (c) Anti-phase and independent oscillations(ω=0.64). (d) Double-mode oscillation (ω=1.09).

Fig. 2: Computer simulation results (ε=0.2, γ=0.2).

Next, we calculate the relationship between the amplitude of the Nth oscillator and the frequency ω. The simulated results for $N = 3, 4$ is shown in Fig. 3.

Figure 4 shows the frequency at the break down of the N-phase synchronization by changing the number of oscillators. From this results, we confirm that the odd number coupling is more stable than even number coupling.



(a) N=3. (b) N=4.

Fig. 3: Relation between amplitude and ω .

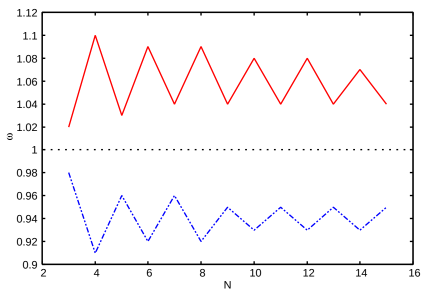


Fig. 4: Frequency for break down of N-phase synchronization (ε=0.2, γ=0.2).

5. Conclusions

In this study, we have investigated synchronization phenomena in a ring of van der Pol oscillators with different frequencies. By carrying out computer simulations, we observed that oscillation of the Nth oscillator stops in some range of the frequency ratio and that others are synchronized as if the Nth oscillator does not exist.