Synchronization in a Ring of van der Pol Oscillators with Different Frequencies

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1. Introduction
In this study, we investigate synchronization phenomena in a ring of van der Pol oscillators with different frequencies. Namely, only the $N$th oscillator has different frequency. By carrying out computer simulations, we observe that oscillation of the $N$th oscillator stops in some range of the frequency ratio and that others are synchronized as if the $N$th oscillator does not exist.

2. Circuit Model
The circuit model is shown in Fig. 1. In the system only the $N$th oscillator has different oscillation frequency.

![Circuit model](image)

Fig. 1: Circuit model.

The normalized circuit equations of the array of oscillators are given as

\[
\begin{align*}
\frac{dx_k}{dt} &= \varepsilon(1-x_k^2) - (y_{ak} + y_{bk}) \\
\frac{dx_N}{dt} &= \omega^2(1-x_N^2) - \omega^2(y_{aN} + y_{bN}) \\
\frac{dy_{ak}}{dt} &= \frac{1}{2}(2x_k - \eta y_{ak} - \gamma(y_{bk} + y_{b(k+1)})) \\
\frac{dy_{bk}}{dt} &= \frac{1}{2}(2x_k - \eta y_{bk} - \gamma(y_{a(k-1)} + y_{bk})) \\
&\quad(k = 1, 2, \ldots, N)
\end{align*}
\]

where

\[y_{a0} = y_{aN}, \quad y_{bN+1} = y_{b1}.\]

It should be noted that $\gamma$ corresponds to the coupling strength and that $\varepsilon$ corresponds to the nonlinearity of oscillators.

3. Synchronization Phenomena
Figure 2 shows observed phenomena for $N = 3$. For the case that all of these oscillations have the same frequency, we can observe that system is synchronized at the three-phase (Fig. 2(a)). When the frequency of the 3rd oscillator is varied, we confirm that oscillation of the 3rd oscillator stops, namely oscillation death appears (Fig. 2(b)). As increasing the frequency of the 3rd oscillator, oscillation of the 3rd oscillator starts again (Fig. 2(c)). However, the 3rd oscillator is not synchronized to the others. Namely, the 3rd oscillator oscillates alone and the others keep anti-phase synchronization. Furthermore, we observe the double-mode oscillation as shown in Fig. 2(d).

![Simulation results](image)

Fig. 2: Computer simulation results ($\varepsilon=0.2, \gamma=0.2$).

Next, we calculate the relationship between the amplitude of the $N$th oscillator and the frequency $\omega$. The simulated results for $N = 3, 4$ is shown in Fig. 3.

![Amplitude vs. frequency](image)

Fig. 3: Relation between amplitude and $\omega$.

![Frequency for break down](image)

Fig. 4: Frequency for break down of $N$-phase synchronization ($\varepsilon=0.2, \gamma=0.2$).

5. Conclusions
In this study, we have investigated synchronization phenomena in a ring of van der Pol oscillators with different frequencies. By carrying out computer simulations, we observed that oscillation of the $N$th oscillator stops in some range of the frequency ratio and that others are synchronized as if the $N$th oscillator does not exist.