2007 IEEE Workshop on Nonlinear Circuit Networks
NCN'07, Tokushima, Japan, Dec 7--8, 2007

Performance of Quadratic Assignment Problem by Hopfield NN with Scale Law

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***Abstract*— In our past study, the solving ability of the Hopfield Neural Network with noise for quadratic assignment problem is investigated. However, even if we injected the noise to the network, the optimal solution cannot occasionally be found. In this study, we propose the method adding noise with scale law to the Hopfield Neural Network to achieve better performance.**

I. Introduction

Hopfield Neural Network (abbr. NN) [1] is one of the important tools of solving combinatorial optimization problems. The global minimum can be searched by energy decent principle of the Hopfield NN. However, in a lot of cases, the network finds a local minimum, and can not escape from there. In order to avoid this critical problem, many researchers proposed the method adding some kinds of noises to the Hopfield NN. Hayakawa and Sawada pointed out the chaos near the three-periodic window of the logistic map gains the best performance as noise [2].

Figure 1 shows the scale-free network model. In the scale-

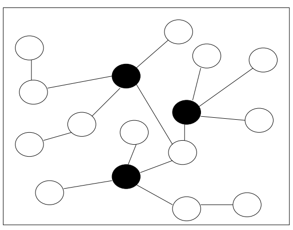


Fig. 1. Scale free network model.

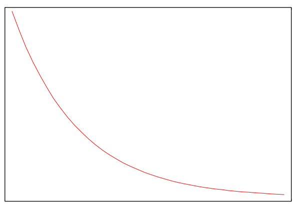


Fig. 2. Scale-free destribution.

free network, a lot of nodes have only few connections, some nodes (marked in black) act as highly connected hubs. So, the small number of important nodes have a lot of connections. This distinction is captured in a more quantitative way by the distribution of the number of links vs. the number of nodes, as shown in Fig. 2. Mathematically, scale-free network are characterized by power law distributions (Fig.2).

In this study, we propose a method to add noise of different amplitudes determined by a scale law. Adding the noise is effective for the solution to escape from the local minimum, but at the same time it may obstruct the energy decent principle. If the noise with a small amplitude is injected to important neurons and the noise with a large amplitude is injected to unimportant neurons, the solution may search only around the global optimum without wandering away. We investigate solving ability of adding the noise with scale law the Hopfield NN for QAP. We confirm that the method is effective to solve QAP by computer simulations.

II. Solving QAP with Hopfield NN

For solving N-element QAP by the Hopfield NN, $N \times N$ neurons are required and the following energy function is defined to fire (i,j)-th neuron at the optimal position:

$$E = \sum_{\mathrm{i,m}=1}^{N} \sum_{\mathrm{j,n}=1}^{N} w_{\mathrm{im;jn}} x_{\mathrm{im}} x_{\mathrm{jn}} + \sum_{\mathrm{i,m}=1}^{N} \theta_{\mathrm{im}} x_{\mathrm{im}}. \quad (1)$$

The neurons are coupled each other with weight between (i,m)-th neuron and (j,n)-th neuron and the threshold of the (i,m)-th neuron are described by:

$$w_{\mathrm{im;jn}} = -2\left\{ A(1-\delta_{\mathrm{mn}})\delta_{\mathrm{ij}} + B\delta_{\mathrm{mn}}(1-\delta_{\mathrm{ij}}) + \frac{C_{\mathrm{ij}} D_{\mathrm{mn}}}{q} \right\} \quad (2)$$

$$\theta_{\mathrm{im}} = A + B \quad (3)$$

where A and B are positive constant, and δ_{ij} is Kroneker's delta. The state of $N \times N$ neurons are asynchronously updated due to the following difference equation:

$$x_{\mathrm{im}}(t+1) = f\left(\sum_{\mathrm{j,n}=1}^{N} w_{\mathrm{im;jn}} x_{\mathrm{im}}(t) x_{\mathrm{jn}}(t) - \theta_{\mathrm{im}} + \beta z_{\mathrm{im}}(t) \right) \quad (4)$$

where f is sigmoidal function.z_{im} is additional noise injected to the network, and β limits amplitude of the noise.

III. Chaos noise

In this section, we describe chaos noise injected to the Hopfield NN. The logistic map is used to generate the chaos noise:

$$\hat{l}_{\mathrm{im}}(t+1) = \alpha_l(t)(1 - \hat{l}_{\mathrm{im}}(t)). \quad (5)$$

Figure III shows an example of the intermittency chaos near the three-periodic window obtained from Eq. (5) for $\alpha_l = 3.8274$.

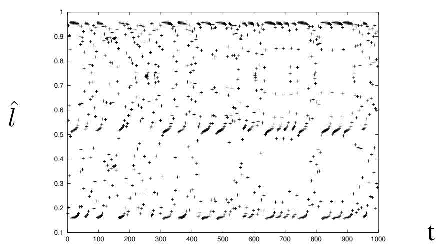


Fig.3. Time series obtained from logistic map for $\alpha_l = 3.8274$.

IV. Noise with Scale Law

We consider that there are some important cities and unimportant cities in a given problem. In other words, there exist some key cities in many problems, whose arrangements should be decided prior to the others to find better solutions. In order to avoid to obstruct the steepest decent principle of the neurons corresponding to such important cities, it may be better not to inject noise with a large amplitude to such neurons. In this study, we propose the method to add noise with scale law based on the concept of the important city and the scale law.

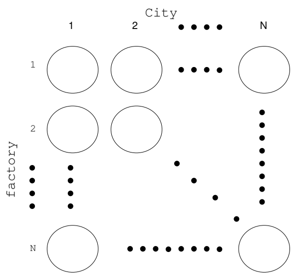
Figure 4 shows the Hopfield NN model for N-element QAP. The neuron of each column expresses the city. The noise with scale law is injected to all the neurons. However, their amplitudes are determined by the column number n_C of the neurons. Namely,

$$\beta = U^{n_C} \tag{6}$$

where U is the constant between 0 and 1, and n_C is the column number.

For the purpose of the comparison, we consider the case that the amplitude is determined by a linear function with the same maximum value.

Further, in order to compare them with the conventional method, the average value of the functions is set up to β_0 of the conventional method.



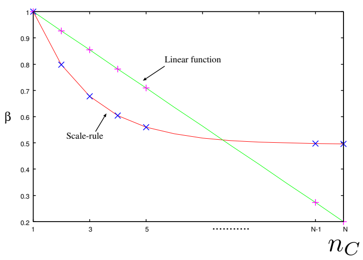


Fig. 4. Neuron of Hopfild NN model for N-element QAP.

Fig.5. Amplitude of noise with scale law.

However, which is important city? Usually, we cannot know how important is each city. Hence, we combine a rearranging the column with the method. In this algorithm, we memorize the column with the first fired neuron. The first fired neuron means the largest output value among the all neurons. So, the column is rearranged to the last (right in Fig. 4). Next, the column with the second fired neuron is the rearranged to the next to the last. In this way, we rearrange the columns in the order of frequency of the firing.

V. Simulation Results

In this section, the simulated results of the Hopfield NN with the noise with scale law for 12-elements of QAP are shown. The problems and parameters are shown as Table 1.

Tables 2 and 3 show the average values of the best solutions obtained during a given iteration numbers for Nug12 and Tai12a, respectively. From these results, the proposed method gains better performance than the conventional method, and the noise with scale law is better than the linear function noise.

TABLE I

Plobrem and parameters.

Problem name	Nug12	Tai12a
Global minimum	578	224416
β_0	0.6	0.5
β_{max}	1.0	1.3
U	0.6	0.5
A,B	0.9	0.9
q	140	16000
ϵ	0.2	0.02

TABLE II

Solving abilities for Nug12.

Iteration	Conventional	Scale law	Linear
1000	632.96	617.96	623.86
2000	623.30	613.32	616.12
3000	619.82	608.50	612.54
4000	616.18	605.78	609.68
5000	613.56	603.02	607.54
6000	612.68	600.94	606.20
7000	611.74	598.84	604.94
8000	610.48	597.82	604.12
9000	610.30	596.74	603.24
10000	609.96	**595.84**	602.36

TABLE III

Solving abilities for Tai12a.

Iteration	Conventional	Scale law	Linear
4000	252624.44	242148.18	244849.92
8000	251337.90	239571.98	242123.36
12000	250291.38	238373.66	240738.44
16000	249638.16	237750.34	239504.80
20000	249520.72	237236.76	238803.02
24000	249495.62	236733.68	238198.66
28000	249458.04	236347.96	237838.76
32000	249335.58	236021.02	237469.98
36000	249228.40	235834.22	237209.98
40000	249225.84	**235549.42**	237121.14

VI. Conclusions

In this study, we proposed the method to add noise with scale law based on the concept of the important city and the scale law. By combining the method with the rearrangement of the columns, we could inject the noise with a large amplitude to unimportant neurons and the noise with a small amplitude to important neurons. By computer simulations, we investigated the solving ability of adding the noise with scale law to the Hopfield NN for QAP.

VII. Acknowledgments

This research is supported by the found of Open Research Center Project from MEXT of Japanese Government (2007-2011).

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