Investigation of Synchronization Phenomena in Cross-Coupled Chaotic Circuits

Yumiko Uchitani and Yoshifumi Nishio
Department of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima, 770-8506 JAPAN
Email: {uchitani, nishio}@ee.tokushima-u.ac.jp

I. Introduction

Synchronization phenomena in complex systems are very good models to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on synchronization phenomena of coupled chaotic circuits are extensively carried out in various fields [1][2]. We have reported an interesting state transition phenomenon observed in simple coupled chaotic circuits [3].

In this study, we investigate the state transition phenomena in detail by computer simulations and circuit experiments.

II. Circuit Model

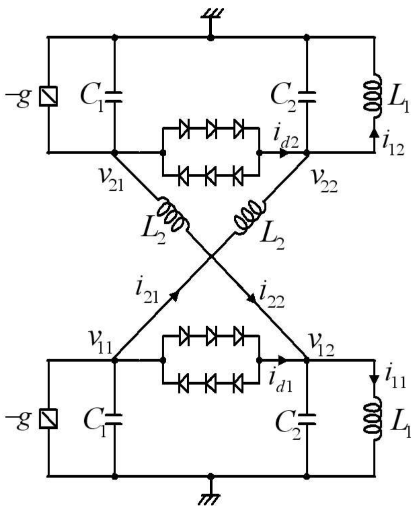


Fig. 1. Circuit model.

Figure 1 shows the circuit, two Shinriki-Mori chaotic circuits [4] are cross-coupled via inductors L_2. By using the following variables and the parameters,

$$\begin{cases} i_{k1} = \sqrt{\dfrac{C_2}{L_1}}Vx_k, \quad i_{k2} = \sqrt{\dfrac{C_2}{L_1}}Vw_k, \\ v_{k1} = Vy_k, \quad v_{k2} = Vz_k, \\ \alpha = \dfrac{C_2}{C_1}, \quad \beta = \sqrt{\dfrac{L_1}{C_2}}G, \quad \gamma = \sqrt{\dfrac{L_1}{C_2}}g, \\ \delta = \dfrac{L_1}{L_2}, \quad t = \sqrt{L_1C_2}\,\tau, \quad \text{``}\cdot\text{''} = \dfrac{d}{d\tau} \qquad (k=1,2) \end{cases} \tag{1}$$

the normalized circuit equations are given as follows.

$$\begin{cases} \dot{x}_k = z_k \\ \dot{y}_k = \alpha\{\gamma y_k - w_k - \beta f\,(y_k - z_k)\} \\ \dot{z}_k = \beta\, f(y_k - z_k) + w_{k+1} - x_k \\ \dot{w}_k = \delta(y_k - z_{k+1}) \\ (k = 1, 2), \end{cases} \tag{2}$$

where f are nonlinear functions corresponding to the $v-i$ characteristics of the nonlinear resistors and are described as follows.

$$f(y_k - z_k) = \begin{cases} y_k - z_k - 1 & (y_k - z_k > 1) \\ 0 & (|y_k - z_k| \le 1) \\ y_k - z_k + 1 & (y_k - z_k < -1) \\ (k = 1, 2). \end{cases} \tag{3}$$

III. State Transition Phenomenon

From the circuit in Fig. 1, we could observe interesting state transition phenomenon. An example of the phenomenon is shown in Fig. 2.

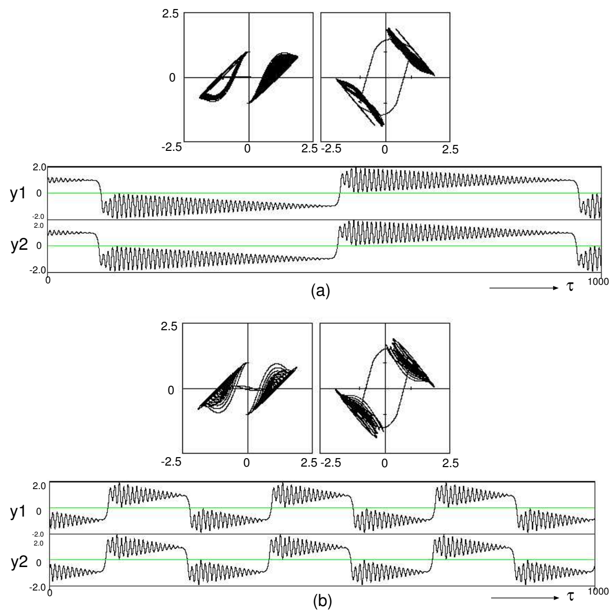
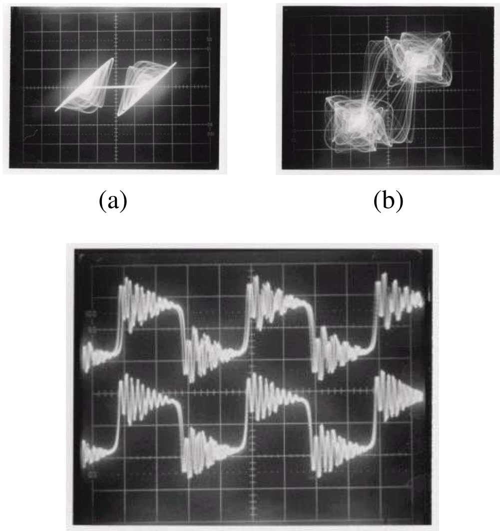


Fig. 2. State transition phenomenon for different coupling parameters (computer calculated result). $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and δ= (a) 0.001, (b) 0.003.

IV. Detailed investigation

In this study, we investigate the above-mentioned state transition phenomena in detail. First, we vary the coupling parameter δ and observe the phenomena. The results are shown in Fig. 2. We can see that the sojourn time between the transitions becomes shorter as increasing the coupling parameter δ.

The corresponding circuit experimental results are shown in Fig. 3.



(a) (b)

(c)

Fig. 3. State transition phenomenon around in-phase synchronization (circuit experimental result). L_1 = 9.93mH, L_2 = 800mH, C_1=32.8nF, and C_2=49.5nF, and g=683mS. (a) Attractor on $v_{11} - v_{12}$ plane. Horizontal and vertical: 1 V/div. (b) Attractor on $v_{11} - v_{21}$ plane. Horizontal and vertical: 1 V/div. (c) Time waveform v_{11} and v_{21}. Horizontal 1.0 ms/div and vertical: 2 V/div.

When the amplitude of the oscillation of the LC resonator approaches to zero, the state transition occurs; namely y abruptly moves toward to the other fixed point quickly. Though the mechanism of this transition cannot be explained very well, we consider that this is because the coupling inductor can be regarded as a short circuit when the oscillation stops. In other words, the coupling inductor plays a role of a kind of switch depending on the oscillation amplitude of the LC resonator.

Figure4 shows the magnification of the time waveform of y. We can see that the switching timing of y_1 and y_2 are synchronized in in-phase, however, small oscillations between the transitions are synchronized in anti-phase.

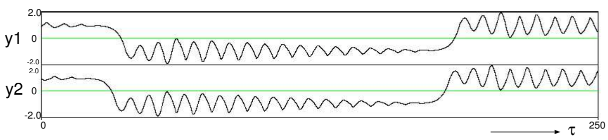


Fig. 4. Magnification of the time waveform around transition. $\alpha = 1.5$, $\beta = 5.0$, $\gamma = 0.2$, and $\delta = 0.003$.

V. Anti-phase state transition

Next, similar state transition can be observed around anti-phase synchronization as shown in Fig. 5. Similar to the in-phase, the sojourn time between the transitions decreases as the coupling δ increases.

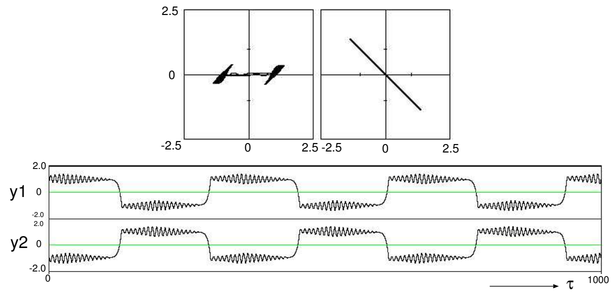


Fig. 5. Anti-phase state transition (computer calculated result). $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.1$, and $\delta = 0.0014$.

VI. Quadrature-phase state transition

We also observed an interesting state transition around quadrature-phase, namely 90 degrees as shown in Fig. 6. The corresponding circuit experimental results are shown in Fig. 11. In this state, the solution moves to the quadrant I, II, III, and IV on the $y_1 - y_2$ plane in order as shown in the figures.

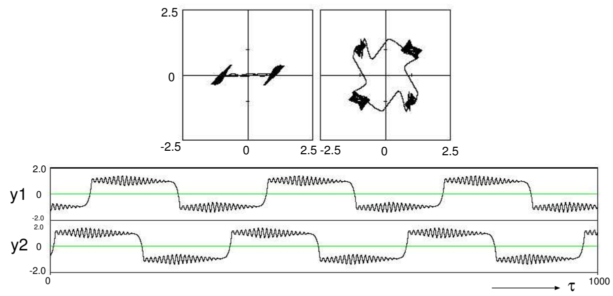
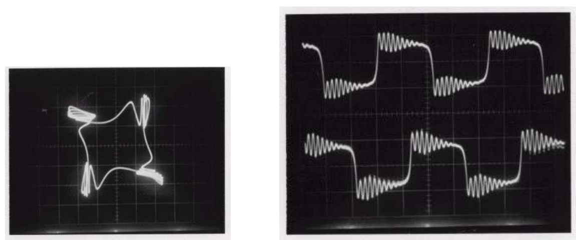


Fig. 6. Quadrature-phase state transition (computer calculated result). $\alpha = 2.0$, $\beta = 4.0$, $\gamma = 0.1$,and $\delta = 0.0014$.



(a) (b)

Fig. 7. Quadrature-phase state transition (circuit experimental result). L_1 = 9.93mH, L_2 = 1.2H, C_1=32.8nF, C_2=49.5nF, and g=495mS. (a) Attractor on $v_{11} - v_{21}$ plane. Horizontal and vertical: 1 V/div. (b) Time waveform v_{11} and v_{21}. Horizontal 1.0 ms/div and vertical: 2 V/div.

VII. Conclusions

In this study, we have investigated interesting state transition phenomenon observed from two Shinriki-Mori chaotic circuits cross-coupled by inductors.

Investigating the coexistence of the states and statistical analysis of the observed phenomena are our important future work as well as more detailed explanation of the mechanism of its generation.

Acknowledgments

This work was partly supported by Yazaki Memorial Foundation for Science and Technology.

References

[1] N. Platt, E.A. Spiegel and C. Tresser, "On-Off Intermittency: A Mechanism for Bursting," Phys. Rev. Lett., vol. 70, no. 3, pp. 279-282, 1993.

[2] Y. Nishio and A. Ushida, "Chaotic Wandering and its Analysis in Simple Coupled Chaotic Circuits," IEICE Trans. Fundamentals, vol. E85-A, no. 1, pp. 248-255, 2002.

[3] Y. Uchitani, R. Imabayashi and Y. Nishio, "State Transition Phenomenon in Cross-Coupled Chaotic Circuits," Proceedings of NOLTA'07, pp. 397-400, 2007.

[4] N. Inaba, T. Saito and S. Mori, "Chaotic Phenomena in a Circuit with a Negative Resistance and an Ideal Switch of Diodes," Trans. of IEICE, vol. E70, no. 8, pp. 744–754, 1987.