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Switching Phenomena of Coexistence Attractors in a Chaotic Map and a Chaotic Circuit

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***Abstract*— Some chaotic systems have two coexistence attractors depending on initial values in same parameters. In these systems, changing parameters cause switching phenomena of two attractors. In this study, we investigate switching phenomena of coexistence attractors in an one-dimensional map and a chaotic circuit. The purpose of this study is a comparison a element of CML with a element of coupled chaotic circuits. Using results of this study, we will investigate the relationship between CML and coupled chaotic circuits.**

I. Introduction

There are many studies of coupled systems. Especially, coupled chaotic systems attract many researchers' attentions because they exhibit a large variety of interesting nonlinear phenomena. One of famous coupled chaotic systems is the Coupled Map Lattice (CML) proposed by Kaneko [1]. CML has been investigated by many researchers and many complex phenomena, for instance, spatio-temporal phenomena, clustering and so on, have been reported. However, this system is a discrete-time system. Some important phenomena including coupled chaotic systems may be lost by time discretization. Therefore, CML should be compared with continuous-time systems. One of the continuous-time system is coupled chaotic circuits. By the reason that characteristics of circuit elements are low price and high quality, you can get elements easily and the repeatability of the experimental results is very high. Additionally, experiments can be carried out in a short time. Coupled chaotic circuits are studied actively. In these studies, phase states or synchronizations are studied mainly.

On the other hands, some chaotic systems have two coexistence attractors depending on initial values in same parameters [2]-[4]. In these systems, changing parameters cause switching phenomena of two attractors. We consider that matching two attractors in coupled systems which have two coexistence attractors means the phenomenon like a very low synchronization. By investigating switching phenomena in the system, it is expected that some interesting phenomena are observed in coupled systems which have two coexistence attractors.

In this study, we investigate switching phenomena of coexistence attractors in an one-dimensional map and a chaotic circuit. The one-dimensional map described by a third-order polynomial function. The chaotic circuit is Shinriki-Mori circuit[2][3]. The purpose of this study is the comparison between chaotic elements of CML and coupled chaotic circuits. Namely, results of this study can make a contribution to the comparison between CML and coupled chaotic circuits.

II. System Models

A. Chaotic Map

The one-dimensional map using in this study is shown as following equation.

$$x_{n+1} = ax_n^3 + cx_n. \tag{1}$$

In order to investigate switching phenomena, a third-order polynomial function are used. Figures 1 show the one-dimensional map using in this study. Two coexistence attractors are observed (red and blue.) In the cases of Fig. 1(a) and (b), same parameters and different initial values are used. By increasing c, switching phenomena are observed in Fig. 1 (c). These are symmetric about the origin. Figure 2 shows the one parameter bifurcation diagram of Eq. (1). The route from periodic orbits via period doubling bifurcations to chaos are observed. Switching phenomena are observed from $\beta = 2.60$ to $\beta = 3.00$. In order to investigate switching phenomena, we define region D^+ and D^- as right and left side attractors of Figs. 1 respectively. Transitional conditions are shown as follows.

$$\begin{array}{l} D^+ \rightarrow D^-\text{: when } x > 0 \text{ and becomes } x < 0. \\ D^- \rightarrow D^+\text{: when } x < 0 \text{ and becomes } x > 0. \end{array} \tag{2}$$

B. Chaotic Circuit

Figure 3 shows the chaotic circuit used in this study. The circuit equation is described as follows.

$$\begin{cases} L\dfrac{di}{dt} &= v_1, \\ C\dfrac{dv_1}{dt} &= -i - i_d, \\ C_0\dfrac{dv_2}{dt} &= gv_2 + i_d, \end{cases} \tag{3}$$

where,

$$i_d = \begin{cases} a(v_1 - v_2 - V_{th}), & v_1 - v_2 > V_{th}, \\ 0, & -V_{th} \le v_1 - v_2 \le V_{th}, \\ a(v_1 - v_2 + V_{th}), & v_1 - v_2 < -V_{th}. \end{cases} \tag{4}$$

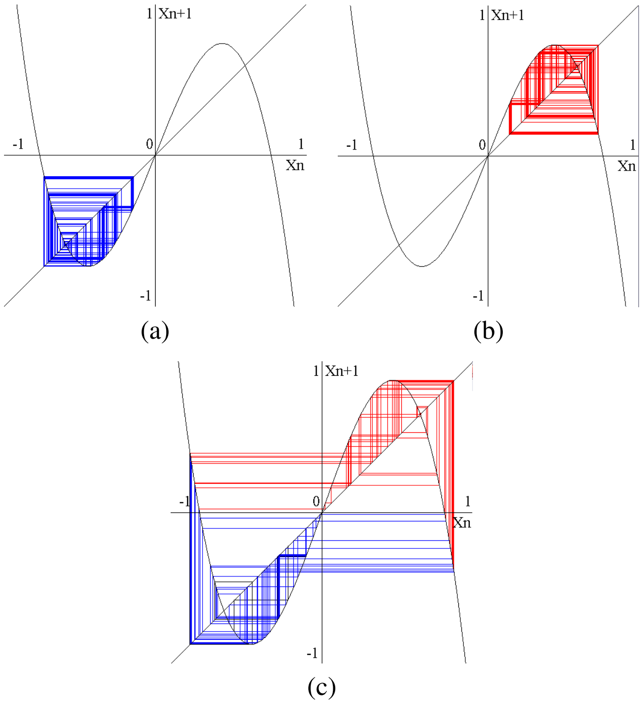

Fig. 1. The one-dimensional map described by a third-order polynomial function. (a) $a = -2.75$ and $c = 2.54$. Initial value of x is -0.4. (b) $a = -2.75$ and $c = 2.54$. Initial value of x is 0.4. (c) $a = -2.75$ and $c = 2.80$.

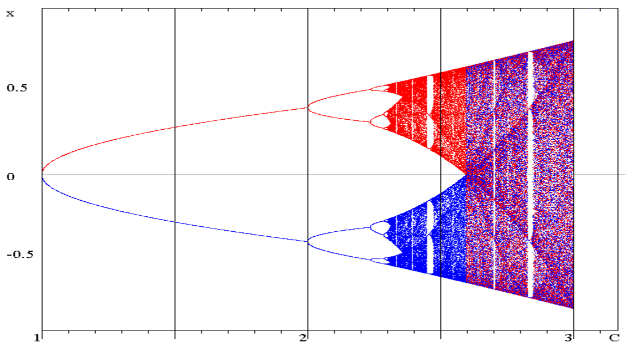

Fig. 2. The one parameter bifurcation diagram of Eq. (1). $a = -2.75$ and $1 < c < 3$. Red: the initial value of x is 0.4. Blue: the initial value of x is -0.4.

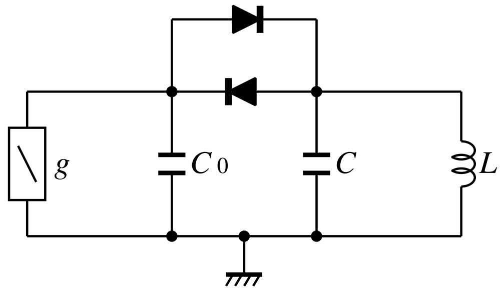

Fig. 3. Shinriki-Mori chaotic circuit.

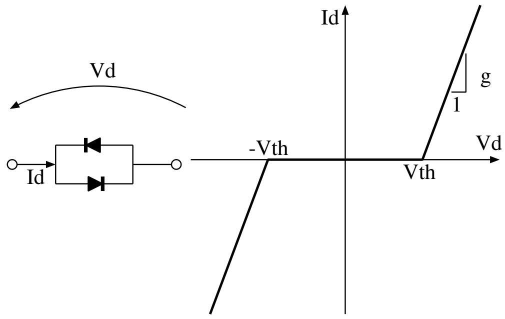

Fig. 4. Diodes model.

Two diodes are modeled as a piece-wise linear function shown in Fig. 4. Changing variables and parameters as follows,

$$x = \frac{1}{V_{th}}\sqrt{\frac{L}{C}} \cdot i, \quad y = \frac{1}{V_{th}} \cdot v_1, \quad z = \frac{1}{V_{th}} \cdot v_2$$
$$\alpha = a\sqrt{\frac{L}{C}}, \quad \beta = g\frac{C}{C_0}\sqrt{\frac{L}{C}}, \quad \gamma = \frac{C}{C_0}, \quad \frac{d}{dt} = \text{"}\cdot\text{"} \tag{5}$$

Normalized circuit equation is described as follows.

$$\begin{cases} \dot{x} = & y, \\ \dot{y} = & -x - f(y-z), \\ \dot{z} = & \beta z + \gamma f(y-z), \end{cases} \tag{6}$$

where,

$$f(y-z) = \begin{cases} \alpha(y-z-1), & y-z > 1, \\ 0, & -1 \le y-z \le 1, \\ \alpha(y-z+1), & y-z < -1. \end{cases} \tag{7}$$

We carry out computer simulations using this equation. Figures. 5 show some computer simulation results. Horizontal axis is z. Vertical axis is y. Two attractors shown in Fig.5 (a) and Fig.5 (b) are observed in same parameters. Here, we define region D^+ and D^- as right and left side attractors of Figs. 5 respectively. Transitional conditions are shown as follows.

$$\begin{array}{l} D^+ \to D^-\text{: when } y-z > -1 \text{ and becomes } y-z < -1. \\ D^- \to D^+\text{: when } y-z < 1 \text{ and becomes } y-z > 1. \end{array} \tag{8}$$

Increasing β, coexistence of attractors are observed shown as Fig. 5 (c). Figure 6 show the one parameter bifurcation diagram of the chaotic circuit. The case that the initial value is 0.11 is shown as red. The case that the initial value is -0.11 is shown as blue. Switching phenomena are observed from $\beta = 0.56$ to $\beta = 0.68$. In this area, phenomena do not depend on the initial value.

III. Switching Phenomena

A. Chaotic Map

Figure 7 shows the magnification of a third-order polynomial function using in this study. In the case of $q < x < p$, switching are occurred. In the case of $0 < x < q$, switching are not occurred. In order to carry out the theoretical analysis, we

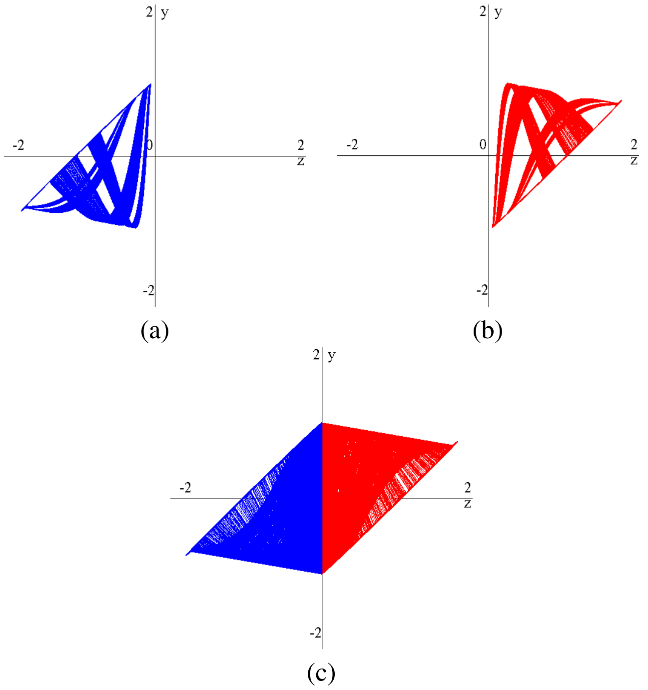

Fig. 5. Computer simulation results of the chaotic circuit. $\alpha = 10.0$ and $\gamma = 2.605$. (a) $\beta = 0.54$. (b) $\beta = 0.54$. (c) $\beta = 0.56$. Differential initial values are used in (a) and (b).

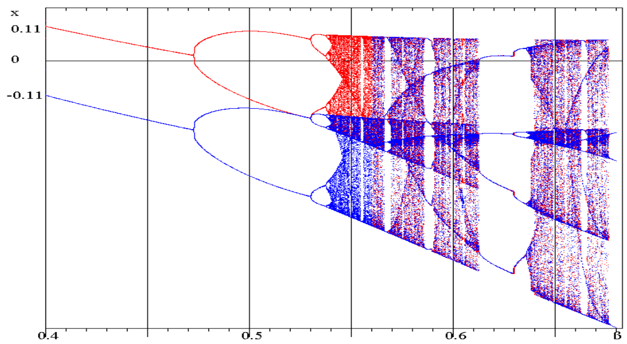

Fig. 6. The one parameter bifurcation diagram of the chaotic circuit. $\alpha = -2.75$ and $0.4 < \beta < 0.68$.

assume that the distribution of x is homogeneous distribution. Using this assumption, we can described the switching rates SR_I as follows.

$$SR_I = 1 - \frac{q}{p} = 1 - \frac{\sqrt{\frac{c}{-a}}}{\frac{2c}{3}\sqrt{\frac{-c}{3a}}}. = 1 - \frac{3\sqrt{3}}{2c}. \tag{9}$$

Figure 8 shows the relationship between parameter c and switching rates. Green line shows SR_I, Blue and Red lines show switching rates calculated by computer simulations. Figures 9 show computer simulation results. In Fig. 8, blue and red lines are the same. Because this map is symmetric about the origin. Some local maximums are observed by periodic orbits shown as Fig. 9 (b) and (d). We consider that the reason of differences between SR_I and switching rates are that the distribution of x is not homogeneous. However, these gradients are similar. We will investigate the distribution of x in this map and improve the SR_I.

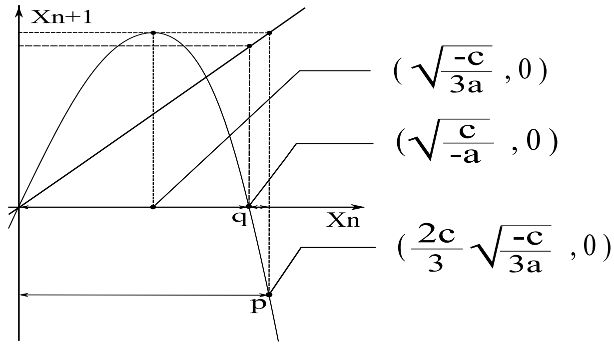

Fig. 7. The magnification of a third-order polynomial function.

B. Chaotic Circuit

Figure 10 shows the relationship between parameter β and switching rates of the chaotic circuit. Horizontal axis shows β and vertical axis shows switching rates. Figures 11 show some computer simulation results. Some periodic orbits (b)(c)(d) are observed. In these areas, switching phenomena are stable states. In another areas, switching rates are increased by increasing β. This system is not symmetric in the origin. Therefore, red and blue lines are not the same. However, very similar results are confirmed. For instance, blue and red lines are corresponding to blue and red in Figs. 5 and Figs. 9.

C. Comparison

Both of them, periodic orbits, period doubling bifurcation, chaos, switching phenomena are observed. Increasing parameter c or β increase switching rates. These results show that two system are similar. Major difference is that the chaotic map is symmetry system. In order to investigate coupling system, we will closer this map to the chaotic circuit.

IV. Conclusions

We have investigated switching phenomena of coexistence attractors in an one-dimensional map and a chaotic circuit. Using obtained results, we will investigate the switching phenomena of CML and coupled chaotic circuits.

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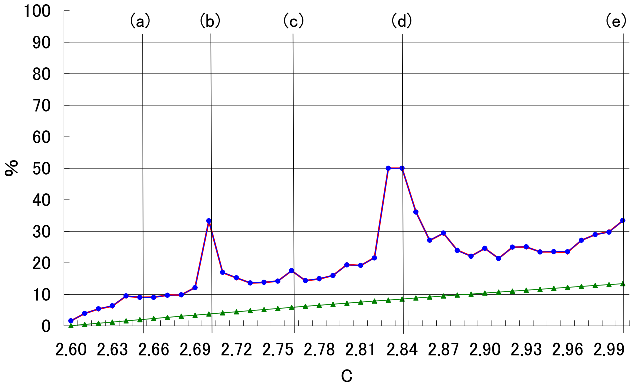


Fig. 8. Switching rates of the chaotic map. $a = -2.75$, $2.60 < c < 3.00$

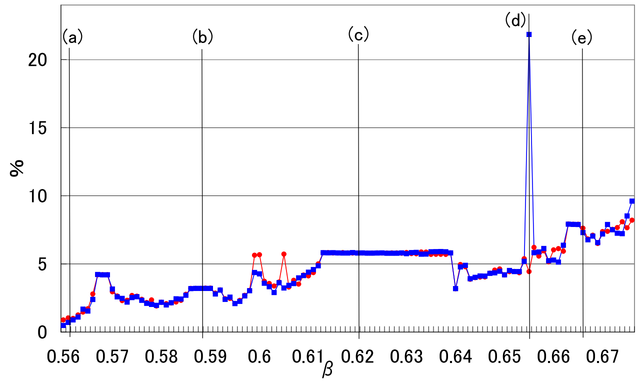
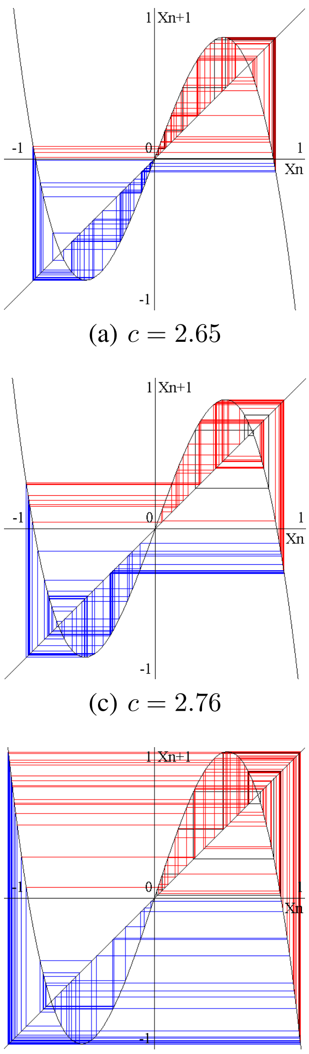


Fig. 10. Switching rates of the chaotic circuit. $\alpha = 10.0$, $\gamma = 2.605$ and $0.56 < \beta < 0.68$.

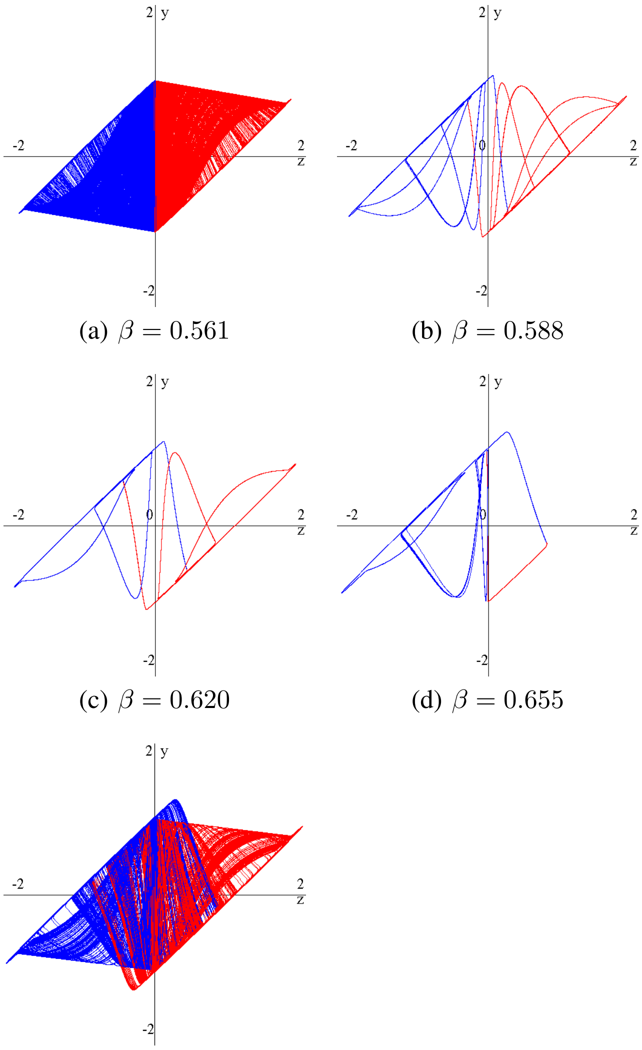


(a) $c = 2.65$ (b) $c = 2.70$

(c) $c = 2.76$ (d) $c = 2.84$

(e) $c = 3.00$

Fig. 9. Computer simulation results of the chaotic map. $a = -2.75$



(a) $\beta = 0.561$ (b) $\beta = 0.588$

(c) $\beta = 0.620$ (d) $\beta = 0.655$

(e) $\beta = 0.666$

Fig. 11. Computer simulation results of the chaotic circuit. $\alpha = 10.0$, $\gamma = 2.605$