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# Wave Propagation Phenomena in Two Rows Cellular Neural Networks Using Two kinds of Templates

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***Abstract*— We have proposed cellular neural networks using two kinds of templates. Our aim is investigation of a new class of complex systems using this CNN. In this study, we investigated the case of two rows for a new class. As a result, wave propagation phenomena can be observed in this system.**

## I. Introduction

Cellular Neural Network (CNN) [1]-[3] is one kind of neural networks. The main characteristic is the local connection. There have been many studies on CNN and many kinds of CNN have been proposed. One of them is two-layer CNN. Two-layer CNN can generate many interesting phenomena. For instance, self-organizing pattern [4], active wave propagation [5] and so on. Investigating these phenomena contributes to understand complex systems and to apply them to engineering systems. In our earlier study [6], we propose CNN using two kinds of templates. In this system, pattern formation and active wave propagation are observed. However, these phenomena are same phenomena observed in Two-layer CNN.

In this study, we propose two rows cellular neural networks using two kinds of templates. The aim of this study is to confirm the special phenomena of cellular neural networks using two kinds of templates.

## II. Cellular Neural Networks Using Two Kinds of Templates

Figure 1 shows a system model the structure of CNN using two kinds of templates. We assume that the system has a two-dimensional $M$ by $N$ array structure. Each cell in the array is denoted as $c(i,j)$, where $(i,j)$ is the position of the cell, where $1 \le i \le M$ and $1 \le j \le N$. The coupling radius is assumed to be one. We use two kinds of templates. Cells having one template are called as cell $\alpha$ and the other are called as cell $\beta$. These two types of the cells are placed as checkered. The state equations of the cells are given as follows:

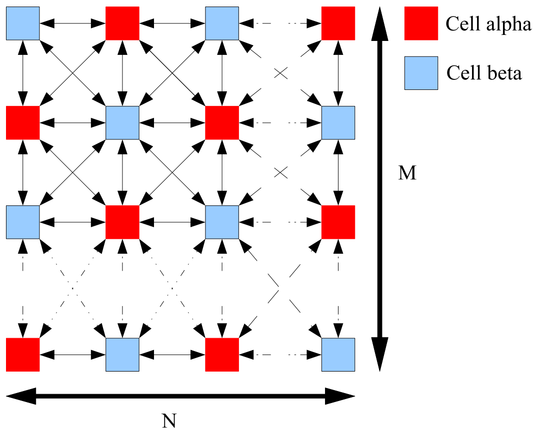


Fig. 1. Structure of CNN using two kinds of templates.

**1:** The case that $i+j$ is an even number.

$$\frac{dx_{ij}}{dt} = -x_{ij} + I_\alpha + \sum_{c(k,l)} A_\alpha(i,j;k,l)y_{kl} + \sum_{c(k,l)} B_\alpha(i,j;k,l)u_{kl} \quad (1)$$

**2:** The case that $i+j$ is an odd number.

$$\frac{dx_{ij}}{dt} = -x_{ij} + I_\beta + \sum_{c(k,l)} A_\beta(i,j;k,l)y_{kl} + \sum_{c(k,l)} B_\beta(i,j;k,l)u_{kl} \quad (2)$$

$A_{\{\alpha\beta\}}(i,j;k,l)y_{kl}$, $B_{\{\alpha\beta\}}(i,j;k,l)u_{kl}$ and $I_{\{\alpha\beta\}}$ are called as the feedback coefficient, the control coefficient and the bias current, respectively.

The output equation of the cell is given as follows:

$$y_{ij} = f(x_{ij}). \quad (3)$$

where,

$$f(x) = 0.5(|x+1| - |x-1|). \tag{4}$$

The variables $u$ and $y$ are the input and output variables of the cell, respectively. $\boldsymbol{A}_\alpha$, $\boldsymbol{B}_\alpha$, $\boldsymbol{A}_\beta$ and $\boldsymbol{B}_\beta$ are 3 times 3 matrices, which can be described to have a similar form to Eq. (5).

$$\begin{pmatrix} A_\alpha(i,j;i-1,j-1) & A_\alpha(i,j;i-1,j) & A_\alpha(i,j;i-1,j+1) \\ A_\alpha(i,j;i,j-1) & A_\alpha(i,j;i,j) & A_\alpha(i,j;i,j+1) \\ A_\alpha(i,j;i+1,j-1) & A_\alpha(i,j;i+1,j1) & A_\alpha(i,j;i+1,j+1) \end{pmatrix} \tag{5}$$

This proposed system is more complex than the normal CNN. This system has a peculiar characteristic in order to investigate a new class of coupled oscillatory systems. Namely, a pair of cell $\alpha$ and cell $\beta$ are needed for a simple oscillation. However, one cell $\alpha$ connects with four neighbor cells $\beta$ and one cell $\beta$ also connects with four neighbor cells $\alpha$. Like this, these cells are sharing a factor of oscillation. This type of connection may be difficult to realize by coupling normal oscillators.

In this study, the number of rows is limited to 2. Because two rows of CNN using two template is corresponding to one row of two layer CNN. By limiting rows, understanding the generated phenomena becomes easy.

## III. Two rows Cellular Neural Networks using two kinds of templates

In the case of one dimensional two layer CNN, wave propagation phenomena are observed using following template [7].

$$\begin{aligned} &\boldsymbol{A}_1 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}, \quad \boldsymbol{A}_2 = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}, \\ &\boldsymbol{C}_1 = \begin{pmatrix} 0 & 10 & 0 \end{pmatrix}, \quad \boldsymbol{C}_2 = \begin{pmatrix} 0.1 & -0.4 & 0.1 \end{pmatrix}, \\ &\boldsymbol{B}_1 = 0, \quad \boldsymbol{B}_2 = 0, \quad I_1 = 0, \quad I_2 = 0, \end{aligned} \tag{6}$$

Couplings of cells are shown in Fig. 2.

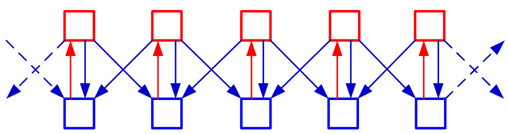

Fig. 2. Coupling of cells in the case of wave propagation phenomena.

We consider reproducing this system using two rows CNN. In our earlier study, we could realize the same structure with two layer CNNs [6]. Because some values of templates are set to 0. However, this template can not realize same structure with two layer CNNs. Therefore, we try to some approaches as follows.

Figure 3 shows coupling of cells in the case of two rows CNN corresponding to Fig. 2. In this case, templates are described as follows.

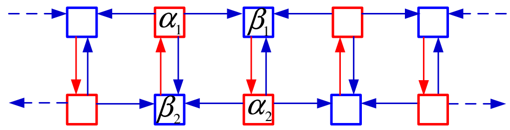


Fig. 3. Coupling of cells in the case of two rows CNN corresponding to Fig. 2.

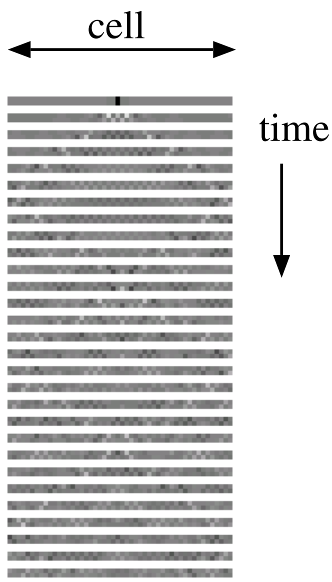


Fig. 4. Computer simulation result of two rows CNN corresponding to Fig. 2.

$$\begin{aligned} &\boldsymbol{A}_{\alpha 1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10 & 0 \end{pmatrix}, \quad \boldsymbol{A}_{\alpha 2} = \begin{pmatrix} 0 & 10 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ &\boldsymbol{A}_{\beta 1} = \begin{pmatrix} 0 & 0 & 0 \\ 0.1 & 1 & 0.1 \\ 0 & -0.4 & 0 \end{pmatrix}, \quad \boldsymbol{A}_{\beta 2} = \begin{pmatrix} 0 & -0.4 & 0 \\ 0.1 & 1 & 0.1 \\ 0 & 0 & 0 \end{pmatrix}, \\ &\boldsymbol{B}_{\alpha 1} = 0, \quad \boldsymbol{B}_{\alpha 2} = 0, \quad \boldsymbol{B}_{\beta 1} = 0, \quad \boldsymbol{B}_{\beta 2} = 0, \\ &I_{\alpha 1} = 0, \quad I_{\alpha 2} = 0, \quad I_{\beta 1} = 0, \quad I_{\beta 2} = 0, \end{aligned} \tag{7}$$

The computer simulation result are shown in Fig. 4. An initial value of center cell are set to 1. initial values of another center cells set 0. Vertical axis shows time. Wave propagation phenomena are observed. The both ends of cells reflect waves. This phenomenon is the same phenomenon observed in the case of Fig. 2 However, this system uses four templates. Two rows CNN using two templates can be realized the similar case to Fig. 3. Namely, coupling of cells are shown as Fig. 5.

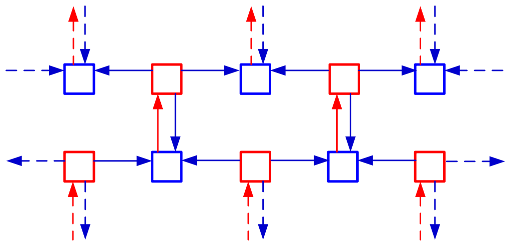

Fig. 5. Coupling of cells in the case of two rows CNN.

In this case, templates are described as follows.

$$\boldsymbol{A}_\alpha = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10 & 0 \end{pmatrix}, \quad \boldsymbol{A}_\beta = \begin{pmatrix} 0 & -0.4 & 0 \\ 0.1 & 1 & 0.1 \\ 0 & 0 & 0 \end{pmatrix}, \quad (8)$$

$$\boldsymbol{B}_\alpha = 0, \quad \boldsymbol{B}_\beta = 0, \quad I_\alpha = 0, \quad I_\beta = 0,$$

Figure 6 shows a computer simulation result in this system. Wave propagation phenomena are not observed. The reason is that only odd-numbered cell are connected. However, self oscillation phenomena are observed as shown in Fig. 6.

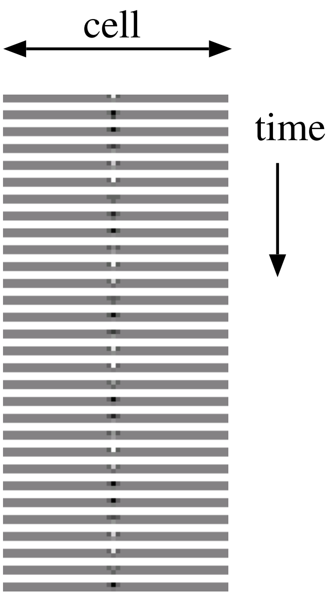


Fig. 6. Computer simulation results of two rows CNN using two templates.

These templates are not symmetric. Therefore, we propose the symmetric template as follows.

$$\boldsymbol{A}_\alpha = \begin{pmatrix} 0 & 10 & 0 \\ 0 & 1 & 0 \\ 0 & 10 & 0 \end{pmatrix}, \quad \boldsymbol{A}_\beta = \begin{pmatrix} 0 & -0.4 & 0 \\ 0.1 & 1 & 0.1 \\ 0 & -0.4 & 0 \end{pmatrix}, \quad (9)$$

$$\boldsymbol{B}_\alpha = 0, \quad \boldsymbol{B}_\beta = 0, \quad I_\alpha = 0, \quad I_\beta = 0,$$

Then, coupling of cells are shown as Fig. 7. This template means that cells of one column are sharing a factor of

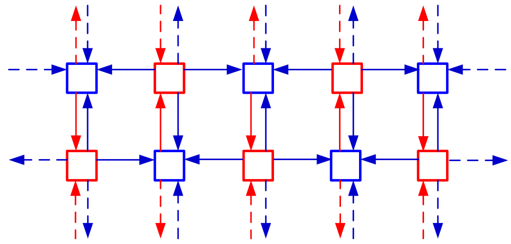

Fig. 7. Coupling of cells in the case of two rows CNN using two template.

oscillation with upper and lower side cells. Figure 8 shows a computer simulation result of two rows CNN. Wave propagation phenomena are observed. This result is same as Fig. 4.

These simulations show that wave propagation are not generated without connecting all cells having one kinds of template with cells having the other kinds of template.

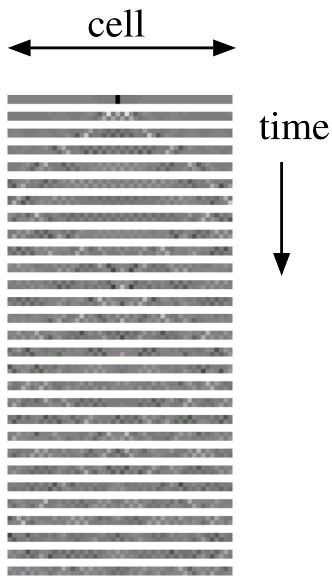


Fig. 8. Computer simulation result of two rows CNN using two template.

## IV. Application of the Connections

Figures 9 show the case of two dimensional CNN using two templates shown in Eq. (9). Figure 9 (a) shows the case that all initial state values are set to 0 without the center cell. The initial state value of the center cell is 1. Figures 9 (b)-(d) show the case that initial state values are set to random numbers. These results are stable states and not the same. These are affected to initial values.

Now, we are investigating CNN using two templates about some kinds of template. Figures 10 show the one of the results. In this simulation, following template is applied.

$$\boldsymbol{A}_\alpha = \begin{pmatrix} 1 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & 1 \end{pmatrix}, \quad \boldsymbol{A}_\beta = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{pmatrix},$$

$$\boldsymbol{B}_\alpha = 0, \quad \boldsymbol{B}_\beta = 0, \quad I_\alpha = 0, \quad I_\beta = 0, \quad (10)$$

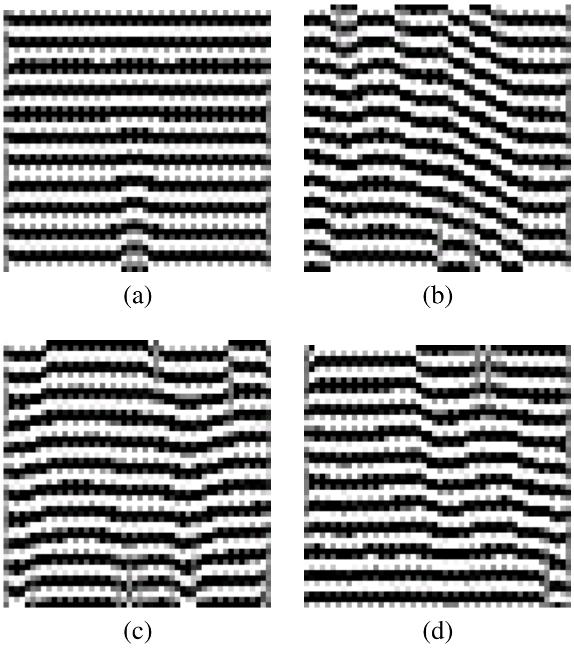


Fig. 9. Computer simulation result of two dimensional CNN using two template. (a) An initial value of center cell are set to 1. initial values of another center cells set 0. (b)-(d) initial values of all cell are set to random values.

Initial state values are set to random numbers as shows in Figs. 10. Clustering phenomena are observed. These clusters do not so move. However, border lines are moving continuously. Some small clusters are disappeared. This phenomenon is also affected to initial values. We can observe some interesting phenomena. We will investigate and analyze these phenomena.

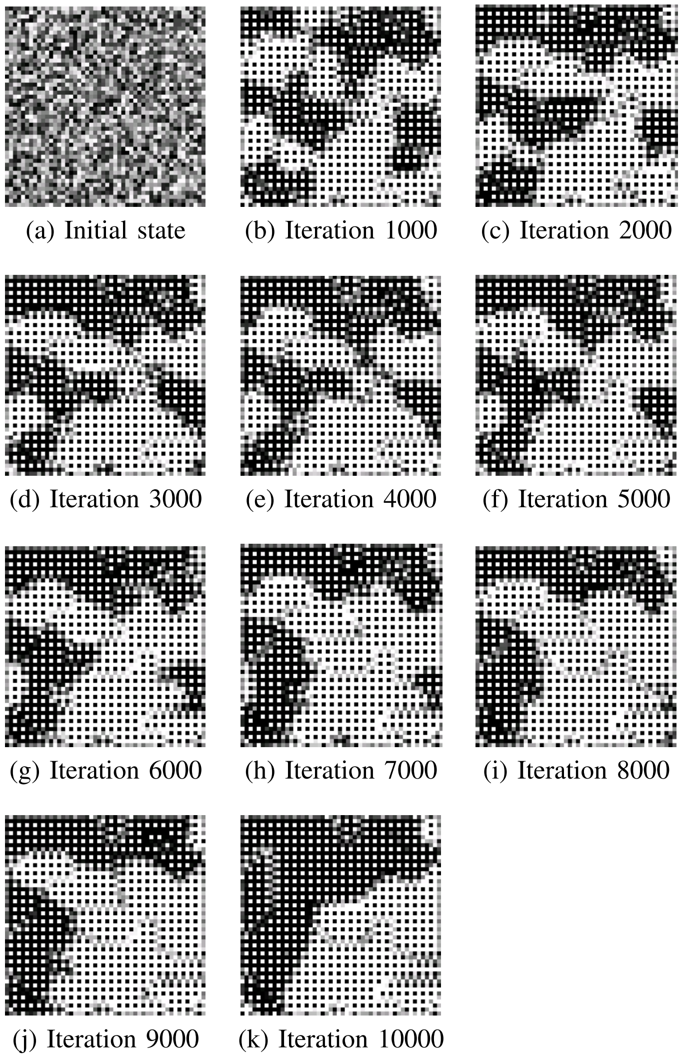


Fig. 10. Computer simulation results of two dimensional CNN using two template shown in Eq. (10).

## V. Conclusions

We have investigated two rows cellular neural networks using two kinds of templates. Wave propagation phenomena are observed. Additionally, some templates of two dimensional cellular neural networks using two templates are proposed. As results of computer simulations, wave propagation phenomena and clustering phenomena are observed.

As a feature research, we will investigate these phenomena in detail.

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