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Synchronization Phenomena in Globally Coupled Maps Including Parameter Mismatches

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1. Introduction

One of famous coupled chaotic systems is the Globally Coupled Map (GCM) proposed by Kaneko [1][2]. GCM has been investigated by many researchers and many complex phenomena, for instance, spatio-temporal phenomena, clustering and so on, have been reported. However, most of the works are found for the system without parameter mismatches in the maps.

In this study, we investigate synchronization phenomena observed from GCM consisting of one-dimensional maps described by a third-order polynomial function including parameter mismatches.

2. System Model

The equation of GCM can be described as follows.

$$x_n(t+1) = (1-\varepsilon)f(x_n(t)) + (\varepsilon/N)\sum_{k=1}^{N} f(x_k(t)) \qquad (1)$$

In this study, a one-dimensional map described by a third-order polynomial function is used for the element of GCM. This map has two control parameters and they affects the strength and the generating region of chaos. Namely, we can give the two different kinds of mismatches to the map by changing the parameters. In order to make the role of the two parameter visible, we define the coordinate of the top of the function as P(p,q). In this case, the third-order function is given as;

$$\begin{aligned} f(x) = \frac{1}{(p^4-2p^2+1)}\Big[&-(2pq+2p)x^3 \\ &+\left\{(3p^2-1)q+3p^2-1\right\}x^2 \\ &+(2pq+2p)x-\left\{(3p^2-1)q+p^4+p^2\right\}\Big] \end{aligned} \qquad (2)$$

3. Computer Simulations

In this article, we show the computer simulated results only for the case that the parameter mismatches are given as follows. The parameter value of q of the map with the index n is given by

$$q_n = \frac{1}{2} + (n-1)\frac{1}{2N} \qquad (n = 1,2,\cdots,N) \qquad (3)$$

where N is the total number of the maps.

$$p = 1.2q - 0.92 \qquad (4)$$

The parameter value of p is given by Eq.(4). Figure 1 shows the rates of synchronization, when the coupling strength ε and the number of the maps N are changed. The red regions correspond to the 100% synchronization. The darkness of the blue regions indicates that the rate of the synchronizations becomes small. We noticed that the synchronization phenomena change around ε =0.2 regardless the number of the maps.

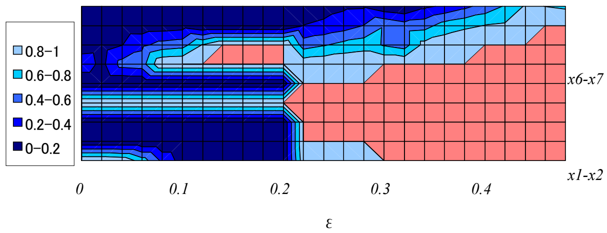

Figure 1: Synchronization rate. Horizontal axis is ε. vertical axes are the differences between xn and xn+1. The red color indicates 100% synchronization.

4. Conclusions

The following results have been obtained.

- Clustering phenomena are observed.
- Synchronization and anti-synchronization phenomena are observed.
- Two different states are changed by the coupling parameter.

The border line of the two states is around ε =0.2.

In our future research, we investigate the detailed influences on the synchronization phenomena of the two parameter which are considered to affect the strength and the generating region of chaos independently.

References

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