17-49

Research on Synchronization Phenomena in Chaotic Oscillators and Noisy Oscillators

Ryo Imabayashi Yoko Uwate Yoshifumi Nishio
(Tokushima University)

1 Introduction

Synchronization is one of typical nonlinear phenomena observed in the field of natural science. Breakdown of synchronization is a kind of cooperative phenomenon for dissipated assembly oscillators and is important to clarify its mechanism for better understanding of higher-dimensional complicated phenomena.

In this study, we investigate the breakdown of synchronization observed from four coupled chaotic oscillators. In order to understand the phenomenon, the model of coupled modfied van der Pol oscillators with noise is considered.

2 Circuit Model

Figure 1 shows the circuit model. In the circuit, four identical chaotic circuits are coupled by one resistor.

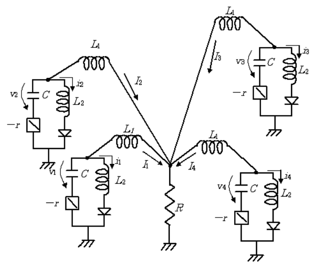

Figure 1: Coupled chaotic oscillators.

The normalized circuit equations of the circuit are described as

$$\begin{cases} \dot{x}_k = \beta(x_k + y_k) - z_k - \gamma \displaystyle\sum_{j=1}^{4} x_j \\ \dot{y}_k = \alpha\{\beta(x_k + y_k) - z_k - f(y_k)\} \\ \dot{z}_k = x_k + y_k, \qquad (k = 1, 2, 3, 4) \end{cases} \tag{1}$$

where $f(y_k) = 0.5(\delta\ y_k + 1 - |\ \delta\ y_k - 1|)$.

Next, we consider four coupled van der Pol oscillators. In order to obtain the waveforms similar to those of the chaotic oscillator, we modify the van der Pol oscillator with the nonlinear resistor whose $v - i$ characteristics are described by the following asymmetric function

$$i_r(v_k) = -g_1 v_k + g_2 v_k^2 + g_3 v_k^3 \quad (g_1, g_2, g_3 > 0). \tag{2}$$

When we add the noise to the voltage amplitude of the modified van der Pol oscillator, the circuit equation of the coupled oscillators are described as

$$\begin{cases} \dot{x}_k = \xi[-y_k + \varepsilon\{(1 + \sigma(\tau))x_k \\ \qquad - \nu((1 + \sigma(\tau))x_k)^2 - ((1 + \sigma(\tau))x_k)^3\}] \\ \dot{y}_k = (1 + \sigma(\tau))x_k - \gamma \displaystyle\sum_{j=1}^{4} y_j. \qquad (k = 1, 2, 3, 4) \end{cases} \tag{3}$$

While when we add the noise to the voltage period of the modified van der Pol oscillator, the circuit equation of the coupled oscillators are described as

$$\begin{cases} \dot{x}_k = (1 + \sigma(\tau))\xi\{-y_k + \varepsilon(x_k - \nu x_k^2 - x_k^3)\} \\ \dot{y}_k = x_k - \gamma \displaystyle\sum_{j=1}^{4} y_j. \qquad (k = 1, 2, 3, 4) \end{cases} \tag{4}$$

where ξ is the parameter added to tune the period of the waveform and $\sigma(\tau)$ is the added noise.

3 Computer Calculated Results

When the coupling parameter γ is relatively large, both the coupled chaotic oscillators and the modified van der Pol oscillators with noise exhibit four phase synchronizations. While for relatively smaller γ, the synchronizations break down and we observe the switchings of phase states. We define this critical coupling parameter as γ_c and investigate how γ_c changes when the strength of chaos or noise increases.

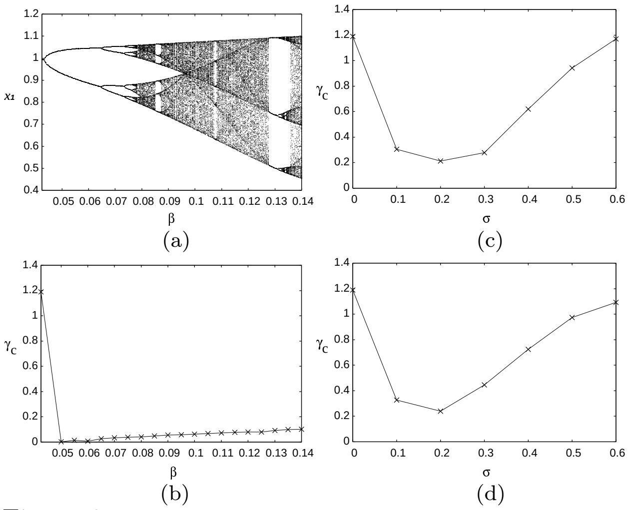

Figure 2: Computer calculated results. (a) One-parameter bifurcation diagram of chaotic oscillator ($\alpha = 7.0$, $\gamma = 0.0$ and $\delta = 100.0$). (b) Breakdown of synchronization of coupled chaotic oscillators ($\alpha = 7.0$ and $\delta = 100.0$). (c) Breakdown of synchronization of modified van der Pol oscillators with noisy amplitude ($\varepsilon = 0.5$, $\xi = 1.07$ and $\nu = 0.1035$). (d) Breakdown of synchronization of modified van der Pol oscillators with noisy period ($\varepsilon = 0.5$, $\xi = 1.07$ and $\nu = 0.1035$).

We noticed that the values of γ_c for the coupled modified van der Pol oscillators are much larger than those for the coupled chaotic oscillators. Although we have to investigate more precisely before concluding, chaotic systems may be synchronized more stably than simple periodic oscillators with noise.

4 Conclusions

In this study, the breakdown of synchronization observed from four coupled chaotic oscillators has been investigated. In order to understand the phenomenon, the model of coupled modified van der Pol oscillators with noise was considered. The simulation results suggested that chaotic systems might be synchronized more stably than simple periodic oscillators with noise.

References

[1] Yoshifumi NISHIO and Akio USHIDA, “Chaotic Wandering and its Analysis in Simple Coupled Chaotic Circuits,” IEICE Transactions on Fundamentals, vol. E85-A, no. 1, pp. 248-255, Jan. 2002.