Statistical Analysis of Clustering in Chaotic Circuits Coupled by an Inductor

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1. Introduction

Spatiotemporal phenomena observed from large-scale coupled chaotic networks have attracted many researchers’ attention and have been studied strenuously by many researchers [1].

In this study, we particularly focus on clustering phenomenon observed from continuous-time real physical systems. We statistically analyze clustering observed from six chaotic circuits coupled by an inductor in detail.

2. Circuit Model

Figure 1 shows a circuit model. In the circuit, \( N \) identical chaotic circuits are coupled by an inductor.

\[
v_d(i_k) = \sqrt[9]{r_d i_k}.
\]

By using the following variables and parameters,

\[
\begin{align*}
 t &= \sqrt{L_1 C \tau}, \quad a = \sqrt{\frac{C}{L_1}}, \quad v_k = a z_k,
\end{align*}
\]

the circuit equations are normalized and described as

\[
\begin{align*}
\dot{x}_k &= \beta(x_k + y_k) - z_k \\
&\quad - \frac{\gamma}{1 + (N-1)\gamma} \sum_{j=1}^{N} \{ \beta(x_j + y_j) - z_j \} \\
\dot{y}_k &= \alpha \{ \beta(x_k + y_k) - z_k - f(y_k) \} \\
\dot{z}_k &= x_k + y_k \\
\end{align*}
\]

where

\[
f(y_k) = \sqrt[9]{y_k}.
\]

3. Clustering Phenomenon

We carried out computer calculations for \( N = 6 \). Figure 2 shows a computer calculated result. From Fig. 2, we can confirm an occurrence of a clustering phenomenon and chaotic changes of the cluster size.

4. Statistical Analysis

The probability distribution of cluster types is shown in Tab. 1. For example, the probability of the cluster type “2-1-1-1-1” is 3.006%. This cluster type means that any two subcircuits are synchronized and four others are asynchronous states and the number of clusters is 5.

<table>
<thead>
<tr>
<th>Cluster type</th>
<th>Probability</th>
<th>Cluster type</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1-1-1-1-1</td>
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<td>3-3</td>
<td>0.25662</td>
</tr>
<tr>
<td>2-1-1-1-1-1</td>
<td>0.03006</td>
<td>4-1-1</td>
<td>0.00033</td>
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<td>4-2</td>
<td>0.000048</td>
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<td>5-1</td>
<td>0.00000</td>
</tr>
<tr>
<td>3-1-1-1-1</td>
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<td>6</td>
<td>0.00000</td>
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<td>3-2-1</td>
<td>0.51865</td>
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</tr>
</tbody>
</table>

5. Conclusions

In this study, we investigated clustering phenomenon observed from six chaotic circuits coupled by an inductor. We analyzed statistical information of the clustering, such as cluster types.

References