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Chaotic Switching of Phase States in Time-Varying Coupled Circuits

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1. Introduction

Synchronization phenomena in complex systems are very interesting to describe various higher-dimensional nonlinear phenomena in the field of natural science. Studies on synchronization phenomena of coupled oscillators are extensively carried out in various fields, physics [1]-[4], biology [5], [6] engineering [7]-[11] and so on. Because many researchers suggest that synchronization phenomena of coupled oscillators have some relations to information processing in the brain. We consider that it is very important to investigate the synchronization phenomena of coupled oscillators to realize a brain computer for the future engineering application.

On the other hand, there are some systems whose dissipation factors vary with time, for example, under the time-variation of the ambient temperature, an equation describing an object moving in a space with some friction and an equation governing a circuit with a resistor whose temperature coefficient is sensitive such as thermistor. However, there are few discussions about coupled oscillators coupling by a time-varying resistor.

In our previous research, we have investigated synchronization phenomena in van der Pol oscillators coupled by a time-varying resistor [12]. We realized the time-varying resistor by switching a positive and a negative resistors periodically. By changing the duty ratio p, we confirmed that the characteristics of the synchronization phenomena changed as follows. First, for smaller p, the two coupled oscillators are synchronized only in anti-phase. Second, for intermediate p, the coexistence of the in-phase and the anti-phase synchronizations can be observed. Finally, for larger p, only the in-phase synchronization can be confirmed. We consider what complex synchronization phenomena can be observed, when chaotic circuits are coupled by time-varying resistor.

In this study, two chaotic circuits coupled by a time-varying resistor are investigated. First, the coexistence of in-phase and anti-phase synchronization are observed. Next, we confirm that interesting synchronization phenomena with switching phase states between the two chaotic circuits can be observed when the strength of coupling parameter R is decreased. Furthermore, the sojourn time of the in-phase and the anti-phase are investigated. We confirm that the sojourn time depends on the frequency and the strength of the time-varying resistor.

2. Circuit Model

Figure 1 shows the circuit model, which is the chaotic version of the circuit investigated in [12]. In the circuit, two identical chaotic circuits are coupled by a time-varying resistor whose characteristics are shown in Fig. 2 [13]. In this study, we consider the case that the function representing the variation of the TVR is the square wave with the angular frequency ω_t and the duty ratio p.

First, the $i-v$ characteristics of the diodes are approximated by two-segment piecewise-linear functions as

$$v_d(i_k) = 0.5(r_d i_k + E - |r_d i_k - E|). \qquad (1)$$

By changing the variables and parameters,

$$I_k = \sqrt{\frac{C}{L_1}} E x_k, \quad i_k = \sqrt{\frac{C}{L_1}} E y_k, \quad v_k = E z_k,$$

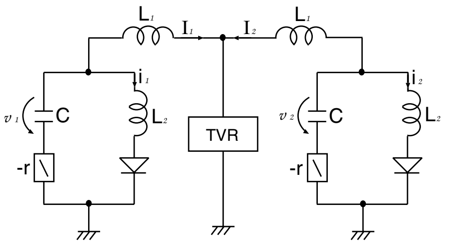


Figure 1: Circuit model (TVR is a Time-Varying Resistor).

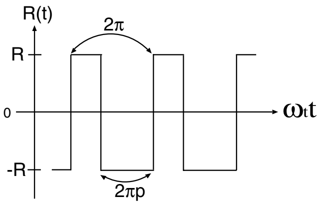


Figure 2: Characteristics of the TVR.

$$t = \sqrt{L_1 C}\tau, \quad \alpha = \frac{L_1}{L_2}, \quad \beta = r\sqrt{\frac{C}{L_1}}, \quad (2)$$
$$\gamma = R\sqrt{\frac{C}{L_1}}, \quad \delta = r_d\sqrt{\frac{C}{L_1}}, \quad \omega = \frac{1}{\sqrt{L_1 C}}\,\omega_t,$$

the normalized circuit equations are given as

$$\begin{cases} \dfrac{dx_k}{d\tau} = \beta(x_k + y_k) - z_k \pm \gamma(x_1 + x_2) \\ \dfrac{dy_k}{d\tau} = \alpha\beta(x_k + y_k) - z_k - f(y_k) \\ \dfrac{dz_k}{d\tau} = x_k + y_k \qquad (k = 1, 2) \end{cases} \quad (3)$$

where the sign of the coupling term changes according to the value of the time-varying resistor. The normalized characteristics of the diodes are given as

$$f(y_k) = 0.5\ (\delta y_k + 1 - |\delta y_k - 1|). \quad (4)$$

3. Synchronization Phenomena

3.1. In-Phase and Anti-Phase Synchronization

We observed that the two coupled oscillators are synchronized in in-phase and anti-phase as shown in Figs. 3 and 4. These two synchronization states can be obtained by giving different initial conditions. The parameters of the chaotic circuits are fixed as $\alpha = 7.0$, $\beta = 0.084$, $\gamma = 0.1$, $\omega = 1.924$ and $p = 0.5$.

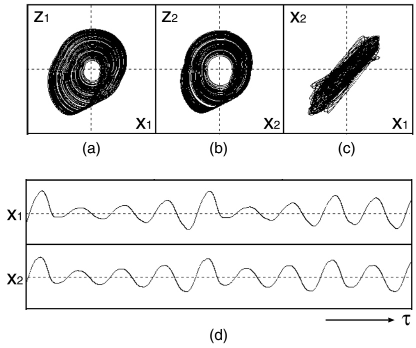


Figure 3: In-phase synchronization (computer simulation results). (a) 1st circuit attractor (x_1 vs z_1). (b) 2nd circuit attractor (x_2 vs z_2). (c) Phase difference (x_1 vs x_2). (d) Time wave form (τ vs x_1 and x_2).

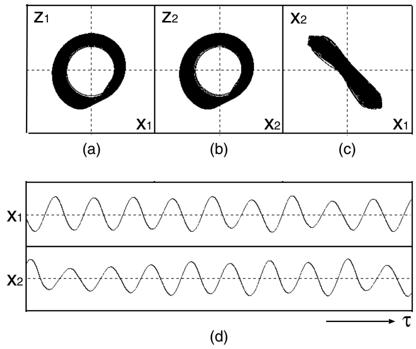


Figure 4: Anti-phase synchronization (computer simulation results). (a) 1st circuit attractor (x_1 vs z_1). (b) 2nd circuit attractor (x_2 vs z_2). (c) Phase difference (x_1 vs x_2). (d) Time wave form (τ vs x_1 and x_2).

We also confirm that the two coupled chaotic circuits are synchronized in in-phase or at anti-phase in circuit experiments as shown in Figs. 5 and 6.

The one parameter bifurcation diagram of x_1 for in-phase and anti-phase synchronization modes are shown in Fig. 7. Figure 8 show the one parameter diagram of the phase difference. We can confirm the coexistence of in-phase and anti-phase synchronizations for $0.055 < \beta < 0.090$.

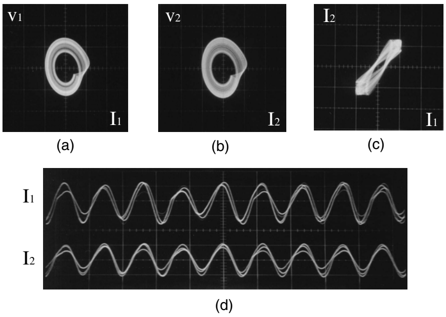


Figure 5: In-phase synchronization (circuit experimental results). (a) 1st circuit attractor (I_1 vs v_1). (b) 2nd circuit attractor (I_2 vs v_2). (c) Phase difference (I_1 vs I_2). (d) Time wave form (t vs I_1 and I_2). $L_1 = 300mH, L_2 = 10mH, C = 33nF, r = 700\Omega$ and $R = 100\Omega$.

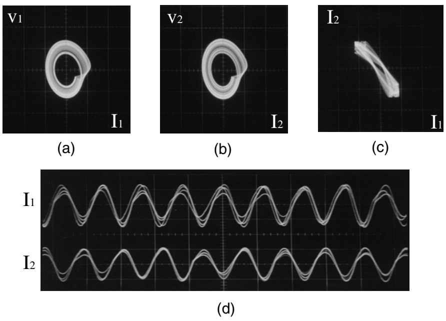
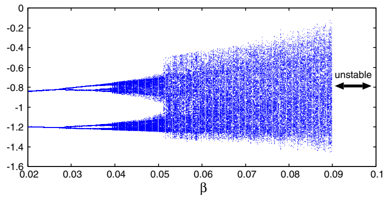
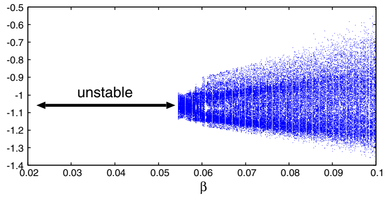


Figure 6: Anti-phase synchronization (circuit experimental results). (a) 1st circuit attractor (I_1 vs v_1). (b) 2nd circuit attractor (I_2 vs v_2). (c) Phase difference (I_1 vs I_2). (d) Time wave form (t vs I_1 and I_2). $L_1 = 300mH, L_2 = 10mH, C = 33nF, r = 700\Omega$ and $R = 100\Omega$.



(a) Bifurcation diagram of x_1 for in-phase synchronization mode.



(b) Bifurcation diagram of x_1 for anti-phase synchronization mode.

Figure 7: One parameter bifurcation diagrams for $\alpha = 7.0$, $\gamma = 0.1$, $\omega = 1.924$. Horizontal axis: β.

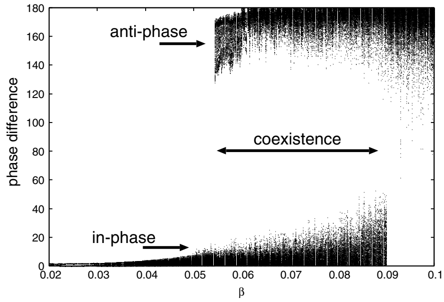


Figure 8: Phase difference.

3.2. Switching Phase States

In this section, we investigate the synchronization phenomena when the strength of the coupling parameter γ is decreased. We can confirm that the switching of the phase states between the in-phase and the anti-phase is observed as shown in Fig. 9. These switching phenomena could not be confirmed in the two van der Pol oscillators coupled by time-varying resistor. The chaotic circuits coupled by time-varying resistor has possibility to generate complex phenomena.

Next, we pay our attention to the sojourn time of the in-phase state and the anti-phase state. We carry out the 30 moving average of the phase difference between two coupled chaotic circuits to distinguish the in-phase state and the anti-phase state more correctly. The simulated result of the moving average of the phase difference is shown in Fig. 10. We define in-phase or anti-phase synchronization, by using the 30 moving average of the phase difference. Namely, when the phase difference is smaller or larger than 90 degrees, the synchronization state of two chaotic circuits are determined to in-phase or anti-phase state.

The frequency distribution of the sojourn time of the synchronization states is investigated. Figure 11 shows the simulated results of the frequency distribution. From these figures, the frequency distribution of the in-phase and the anti-phase is similar when the parameters of the chaotic circuits are set as follows: $\alpha = 7.0$, $\beta = 0.084$, $\gamma = 0.095$ and $\omega = 1.924$.

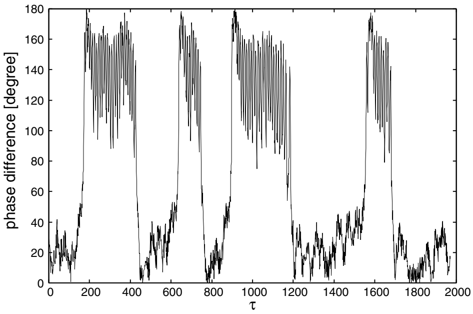


Figure 9: Switching phase states ($\gamma = 0.095$).

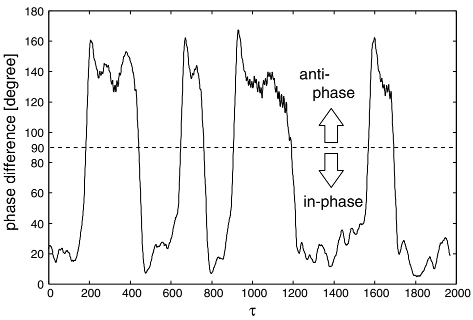
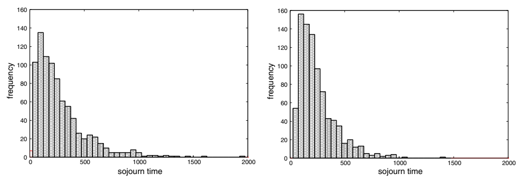


Figure 10: Moving average of the phase states.

Furthermore, the average sojourn time of the in-phase state and the anti-phase state when the frequency ω of the time-varying resistor is changed. The simulated result is shown in Fig. 12. The horizontal axis is frequency ω and the vertical axis is average sojourn time. The average sojourn time of the in-phase state increase by increasing ω. On the other hand, the average sojourn time of the anti-phase state is almost constant. From these results, we can see that the anti-phase state does not depends on the frequency ω of the time-varying resistor. When ω is smaller than 1.908, two chaotic circuits coupled by time-varying resistor do not synchronous neither the in-phase state nor the anti-phase state. And ω is larger than 1.930, the only in-phase state can be occurred.

Some example of the switching synchronization state and the frequency distribution when ω are set to 1.922 and 1.926 are shown in Figs. 13 and 14, respectively. In the case of $\omega = 1.922$, the sojourn time of the in-phase state is longer than the anti-phase state (Fig. 13). In the case of $\omega = 1.926$, the sojourn time of the anti-phase state is longer than the in-phase state (Fig. 14).



(a) In-phase. (b) Anti-phase.

Figure 11: Frequency distribution of sojourn time.

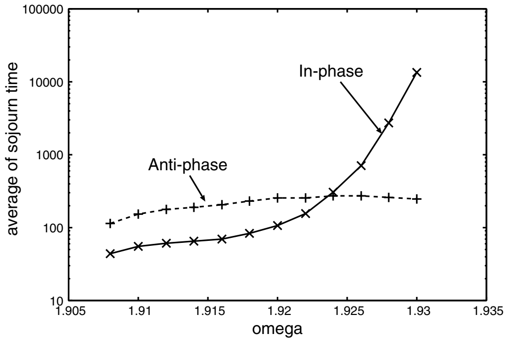


Figure 12: Average sojourn time in dependence on ω.

Next, we investigate synchronization in dependence on the coupling strength γ. The average sojourn time of the in-phase state and the anti-phase state is shown in Fig. 15. The average sojourn time of the in-phase state increase by increasing γ. On the other hand, the average sojourn time of the anti-phase state is almost constant. From these results, we can see that the anti-phase state does not depends on γ as well as frequency ω.

Finally, we investigate synchronization in dependence on the parameter mismatch. The parameter β of the other side chaotic circuit is fixed as $\beta = 0.084$. We change the parameter β_2 of the one side chaotic circuit from $\beta_2 = 0.085$ to $\beta_2 = 0.078$. The average sojourn time of the in-phase state and the anti-phase state is shown in Fig. 16. The average sojourn time of the both in-phase state and anti-phase increase by increasing parameter mismatch.

Some example of the switching synchronization state and the frequency distribution when β_2 are set to 0.083 and 0.080 are shown in Figs. 17 and 18, respectively. In the case of $\beta_2 = 0.083$, the sojourn time of the both in-phase state and anti-phase sate are short (Fig. 17). In the case of $\omega = 0.080$, the sojourn time of the both in-phase state and anti-phase sate become to long (Fig. 18).

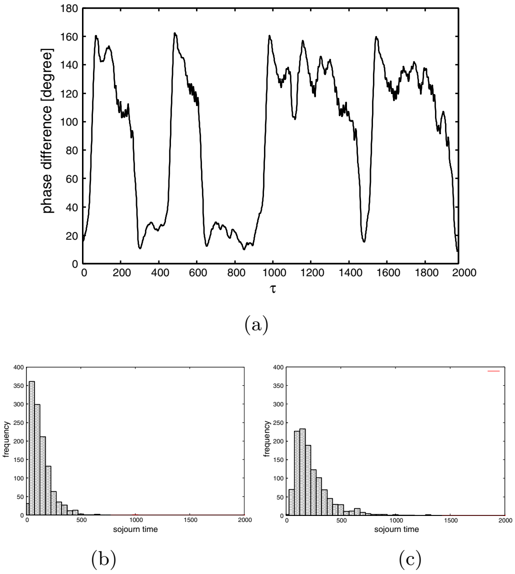


Figure 13: Switching phase state for $\omega = 1.922$. (a) Moving average of switching synchronization state. (b) Frequency distribution of sojourn time (in-phase). (c) Frequency distribution of sojourn time (in-phase).

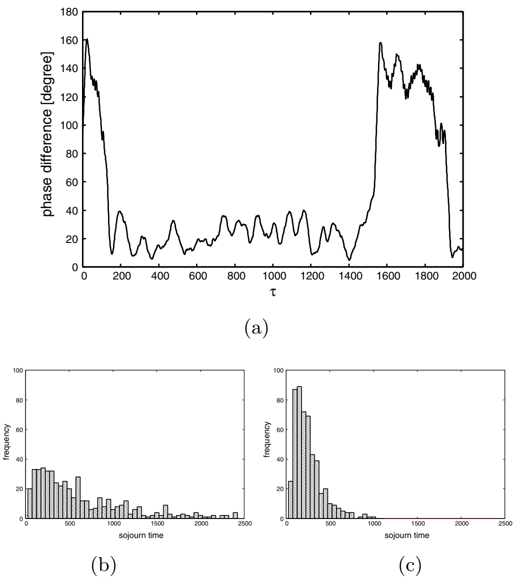


Figure 14: Switching phase state for $\omega = 1.926$. (a) Moving average of switching synchronization state. (b) Frequency distribution of sojourn time (in-phase). (c) Frequency distribution of sojourn time (in-phase).

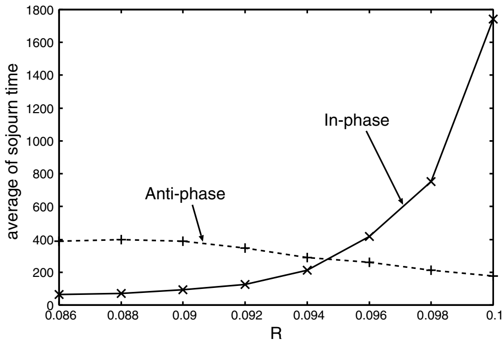


Figure 15: Average sojourn time in dependence on γ.

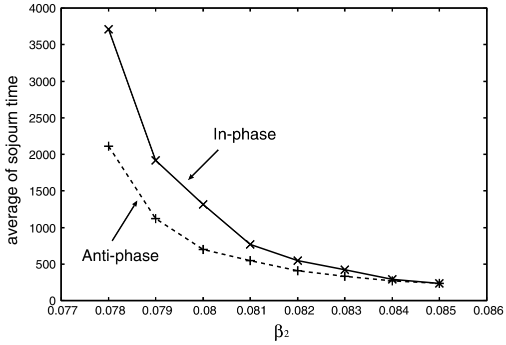


Figure 16: Average sojourn time in dependence on parameter mismatch.

4. Conclusions

In this study, we have investigated phase differences in two chaotic circuits coupled by a time-varying resistor when the frequency of the time-varying resistor is changed. By carrying out computer calculations, we can confirmed that the synchronization phenomena of the two coupled chaotic circuits is depend on the frequency of the time-varying resistor. Further, we investigated the sojourn time of the synchronization state when the frequency ω and coupling strength γ are changed. The average sojourn time of the anti-phase has no influence for changing ω and γ.

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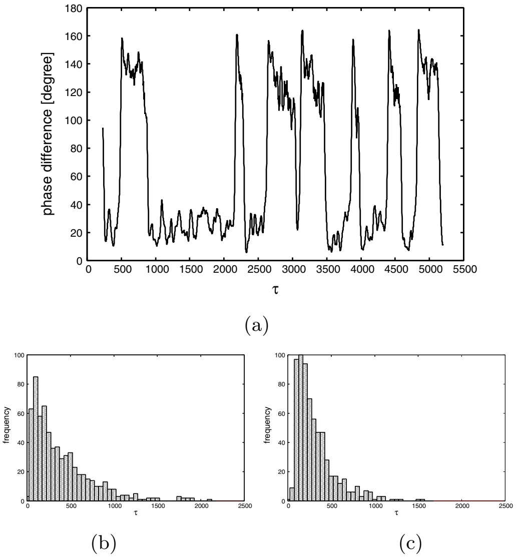

Figure 17: Switching phase state for $\beta_2 = 0.083$. (a) Moving average of switching synchronization state. (b) Frequency distribution of sojourn time (in-phase). (c) Frequency distribution of sojourn time (in-phase).

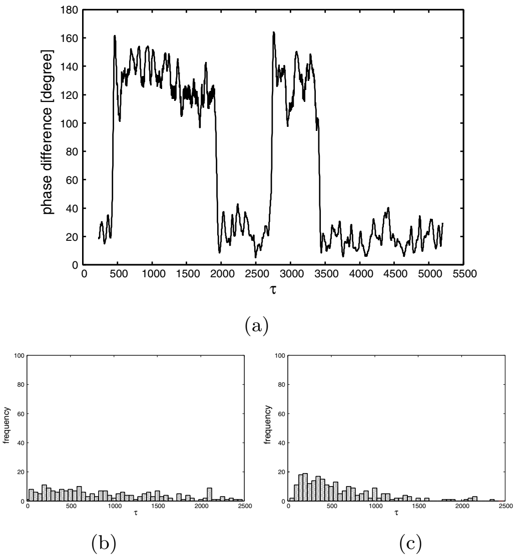

Figure 18: Switching phase state for $\beta_2 = 0.080$. (a) Moving average of switching synchronization state. (b) Frequency distribution of sojourn time (in-phase). (c) Frequency distribution of sojourn time (in-phase).

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