

Research on Synchronization of Chua's Circuit with Transmission Lines

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Abstract—In this study, we investigate synchronization of two Chua's circuits with a lossless transmission lines. Two circuits with transmission lines placed in parallel cause the cross talk phenomena.

By computer simulations, we investigate the synchronization of the two chaotic circuits linked by the cross talk. .

I. INTRODUCTION

Recently, several researchers have investigated synchronization of chaotic circuits. Chua's circuit is one of the simplest autonomous chaotic circuits. Chua's circuit consists of a resistor, a capacitor, a nonlinear resistor and an LC resonator.

By replacing parallel LC resonator in original Chua's circuit, we can consider it is "time-delayed Chua's circuit".[1] We consider two "time-delayed Chua's circuits" placed in parallel. Transmission lines placed in parallel cause the cross talk phenomena.

II. CROSS TALK FROM OPPOSITE DIRECTION

In our previous research, we reported that two Chua's circuits with lossless transmission lines could be synchronized.[2] We model the lossless transmission lines by LC ladder circuits with finite numbers of lumped elements as shown in Fig. 1. We carry out computer simulation for this models by using the 4th order Runge-Kutta method. In this simulation, we consider only the case that the two Chua's circuits are identical.

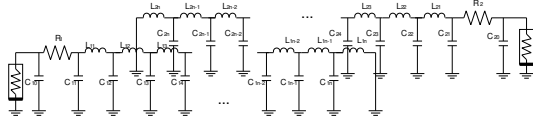


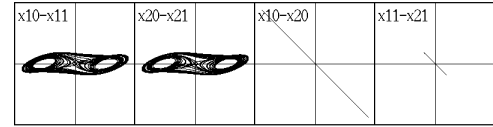
Fig. 1. Discrete model by LC ladder circuits with finite number of lumped elements.

We could obtain the normalized circuit equations as follows;

$$\begin{aligned}
 \dot{x}_{j0} &= \frac{C_{j0}}{C_{j1}} x_{j1} - x_{j0} - f(x_{j0}) \\
 \dot{x}_{j1} &= \alpha_{j1} (y_{j1} - x_{j1} + x_{j0}) \\
 \dot{x}_{jk} &= \alpha_{jk} (y_{jk} - y_{j(k-1)}) \\
 \dot{y}_{j1} &= \gamma_{j1} (x_{j2} - x_{j1}) - \beta (-x_{(j+1)n}) \\
 \dot{y}_{j2} &= \gamma_{j2} (x_{j3} - x_{j2}) - \beta (x_{(j+1)n} - x_{(j+1)(n-1)}) \\
 &\vdots \\
 \dot{y}_{jl} &= \gamma_{jl} (x_{j(l+1)} - x_{jl}) - \beta (x_{(j+1)2} - x_{j1})
 \end{aligned} \tag{1}$$

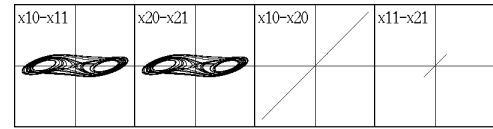
where $x_{3k} = x_{1k}$, $x_{j(n+1)} = 0$, $i_{31} = i_{11}$, ($k=2, 3, \dots, n$), ($l=1, 2, \dots, n$) and ($j=1, 2$),

$$\begin{aligned}
 f(x_{j0}) &= c_j x_{j0} + 0.5(a_j - b_j)(|x_{j0} + 1| - |x_{j0} - 1|) \\
 &\quad + 0.5(b_j - c_j)(|x_{j0} + d_j| - |x_{j0} - d_j|),
 \end{aligned} \tag{2}$$



$\beta = 6.5$.

Fig. 2. Simulation results of cross talk via mutual inductor M .



$\beta = -6.5$.

Fig. 3. Simulation results of cross talk via mutual inductor $-M$.

It is interesting to observe that the two circuits can be synchronized even if they are placed from the opposite direction.

III. RESEARCH

We define equations as follows;

$$\begin{aligned}
 p_k &= x_{1k} - x_{2k} \\
 q_l &= y_{1l} - y_{2l}
 \end{aligned} \tag{3}$$

we can obtain as follows equation

$$\begin{aligned}
 \dot{p}_0 &= \frac{C_0}{C_1} p_1 - p_0 - f(p_0) \\
 \dot{p}_1 &= \alpha (q_1 - p_1 + p_0) \\
 \dot{p}_k &= \alpha (q_k - p_{k-1}) \\
 \dot{q}_1 &= \gamma (q_2 - q_1) - \beta (q_n) \\
 \dot{q}_2 &= \gamma (q_3 - q_2) - \beta (q_{n-1} - q_n) \\
 &\vdots \\
 \dot{q}_l &= \gamma (q_{l+1} - q_l) - \beta (x_1 - x_2)
 \end{aligned} \tag{4}$$

This equation shows existence of the solution on the synchronous plane because $p_k = q_l = 0$ satisfy Eq. (4).

IV. CONCLUSIONS

By computer simulations, we have investigated various interesting phenomena related with chaos synchronization. We confirmed that the existence of the solution on the synchronous plane. In our future work, we investigate the stability of the synchronous plane.

REFERENCES

- [1] J. Kawata, Y. Nishio and A. Ushida, "Analysis of Chua's Circuit with Transmission Line," *IEEE Transactions on Circuits and Systems I*, vol. 44, no. 6, pp. 556-558, 1997.
- [2] Y. Nakaaji and Y. Nishio, "Synchronization of Chaotic Circuit Linked by Cross Talk," *Proceedings of International Symposium on Nonlinear Theory and its Applications*, pp. 43-46, Sep. 2006